Accelerazione

$$\vec{a}(t) = \frac{d\vec{v}(t)}{dt} = \frac{d^2\vec{r}(t)}{dt^2}$$

$$a_x(t) = \frac{dv_x(t)}{dt} = \frac{d^2x(t)}{dt^2}$$

$$a_y(t) = \frac{dv_y(t)}{dt} = \frac{d^2y(t)}{dt^2}$$

$$a_z(t) = \frac{dv_z(t)}{dt} = \frac{d^2z(t)}{dt^2}$$

Dimensioni fisiche

$$[a] = \frac{[v]}{[t]} = \frac{[l]}{[t]^2} \quad \rightarrow \quad \text{S.I.} \ \frac{m}{s^2}$$

Valori tipici

Accelerazione di gravità sulla superficie terrestre:

$g=9.8\ m/s^2$

Accelerazione automobile:

"da 0 a 100 km/h in 10 s"

$$\frac{100\ {}^{km}\!/_{h}}{10\ s} = \frac{\frac{100}{3.6}}{10}\ \frac{m}{s} \approx 2.8\,\frac{m}{s^2}$$

Accelerazione di un razzo alla partenza:

$\approx 5\text{-}8\ g \quad \approx 50\text{-}80\ m/s^2$

Ultracentrifuga : $\approx 10^5\ g \quad \approx 10^6\ m/s^2$

Es: moto rettilineo uniforme

$$\vec{r}(t) = \{v_{ox}t + x_o, v_{yo}t + y_o, v_{zo}t + z_o\}$$

$$\vec{v}(t) = \frac{d\vec{r}(t)}{dt} = \{v_{ox}, v_{yo}, v_{zo}\}$$

$$\vec{a}(t) = \frac{d\vec{v}(t)}{dt} = \frac{d^2\vec{r}(t)}{dt} = \{0,0,0\} \equiv \vec{0}\ (0)$$

Velocità costante = accelerazione nulla

Un altro esempio molto importante:

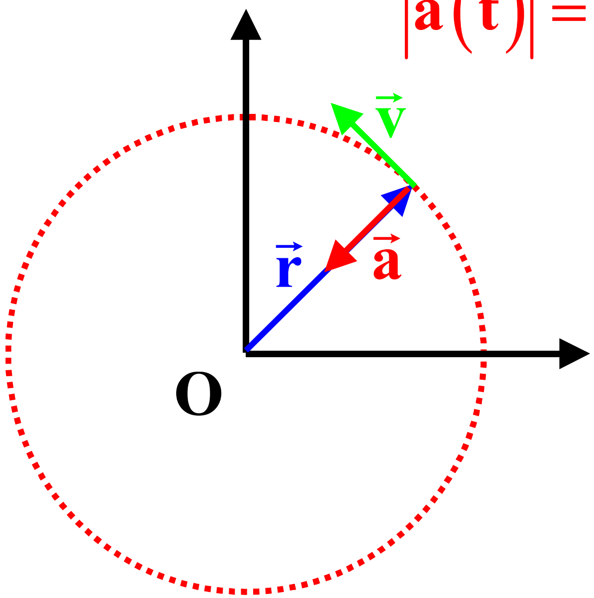
Il moto circolare (uniforme)

$$\vec{r}(t)=\{r_o Cos(\omega t),\ r_o Sin(\omega t),\ 0\} \quad |\vec{r}(t)|=r_o$$

$$\vec{v}(t)=\{-\omega\, r_o Sin(\omega t),\ \omega\, r_o Cos(\omega t),\ 0\} \quad |\vec{v}(t)|=\omega r_o$$

$$\vec{a}(t)=\frac{d\vec{v}(t)}{dt}=\{-\omega^2\, r_o Cos(\omega t),\ -\omega^2\, r_o Sin(\omega t),\ 0\}=-\omega^2\vec{r}(t)$$

$$|\vec{a}(t)|=\sqrt{(\omega^2\, r_o)^2 Cos^2(\omega t)+\ (\omega^2\, r_o)^2 Sin^2(\omega t)}=\omega^2 r_o$$



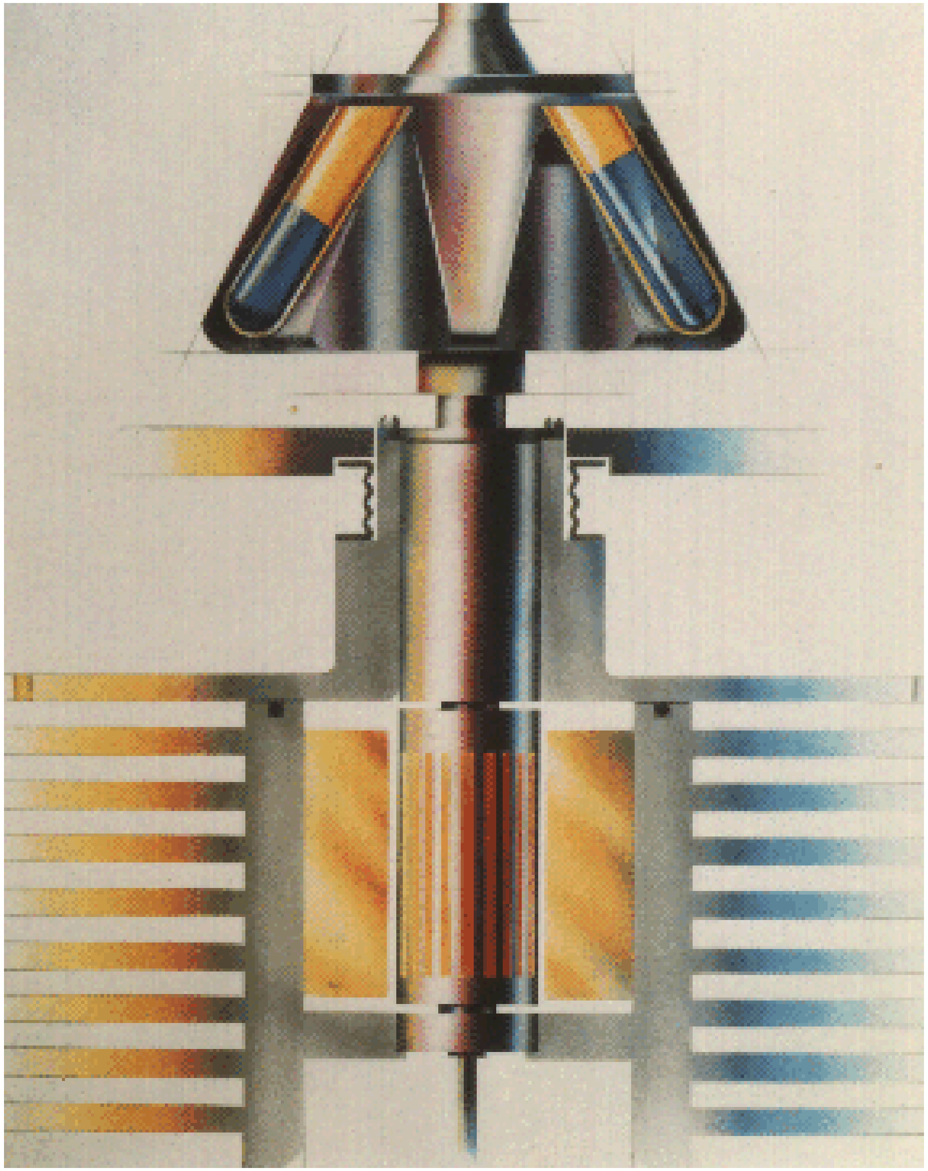
Accelerazione "centripeta"

$$\vec{r}\cdot\vec{v}=0 \quad \vec{v}\cdot\vec{a}=0$$

Ultracentrifuga: $\omega \approx 2\pi 1000\, rad/s;$

$$r_o \approx 0.1m;\ \omega^2 r_o \approx 4\times 10^6 \frac{m}{s^2}$$

Ultracentrifuga Preparativa (Beckmann Coulter)



Spins up to 8 x 6.5 mL tubes up to 802,400 x g @ 100,000 rpm in the XL-100K

Es: moto uniformemente accelerato

$$\vec{r}(t) =$$

$$\left\{ x_o + v_{xo} t + \frac{1}{2} a_{xo} t^2, y_o + v_{yo} t + \frac{1}{2} a_{yo} t^2, z_o + v_{zo} t + \frac{1}{2} a_{zo} t^2 \right\}$$

$$\vec{v}(t) = \frac{d\vec{r}(t)}{dt} = \left\{ v_{xo} + a_{xo} t, v_{yo} + a_{yo} t, v_{zo} + a_{zo} t \right\}$$

$$\vec{a}(t) = \frac{d\vec{v}(t)}{dt} = \left\{ a_{xo}, a_{yo}, a_{zo} \right\} = \vec{a}_o$$

Es: moto rettilineo uniformemente accelerato con partenza da fermo

$$x(t) = \frac{1}{2} a_o t^2$$

$$z(t) = y(t) = 0 \qquad v_z(t) = v_y(t) = 0 \qquad a_z(t) = a_y(t) = 0$$

$$v_x(t) = a_o t$$

$$a_x(t) = a_o$$

Es: moto rettilineo uniformemente accelerato “in verticale”

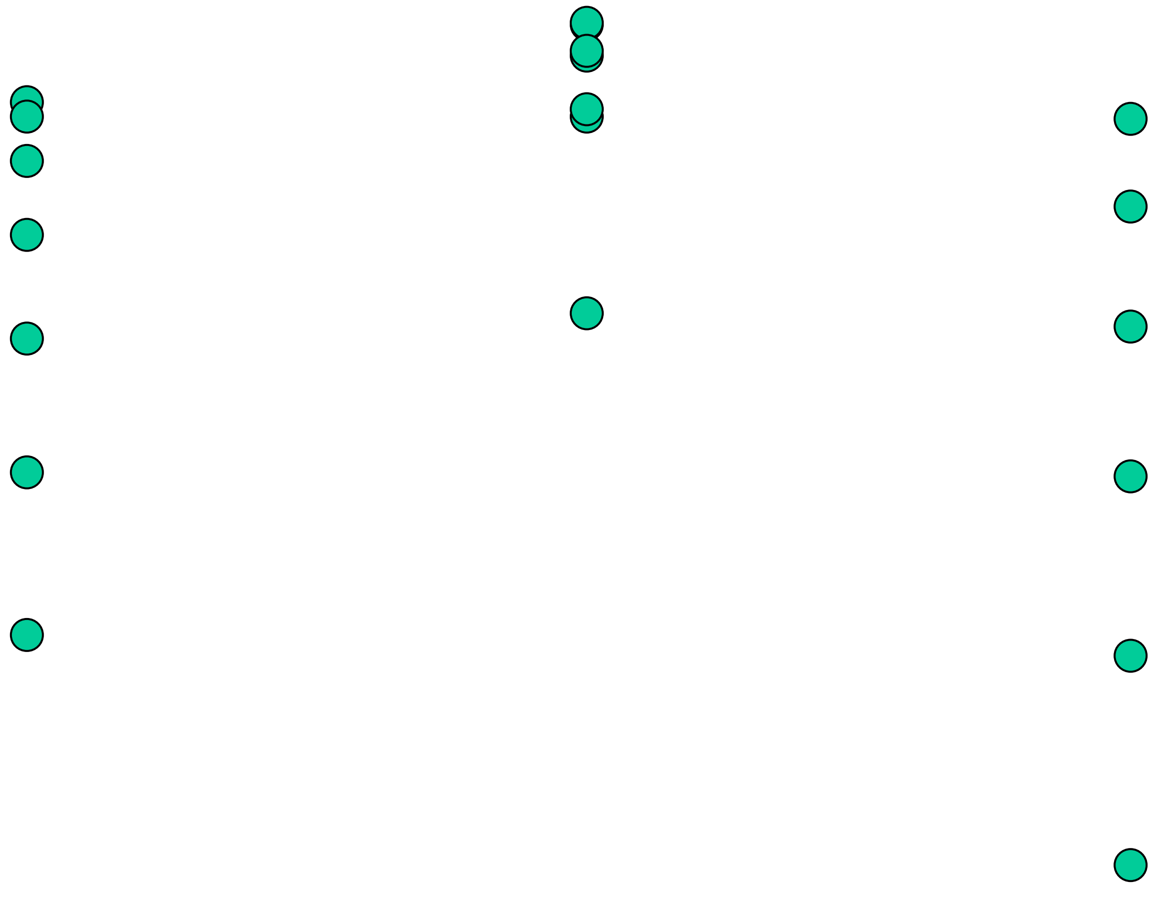
$$z(t) = z_o + v_o t + \frac{1}{2} a_o t^2$$

$$x(t) = y(t) = 0 \qquad v_x(t) = v_y(t) = 0 \qquad a_x(t) = a_y(t) = 0$$

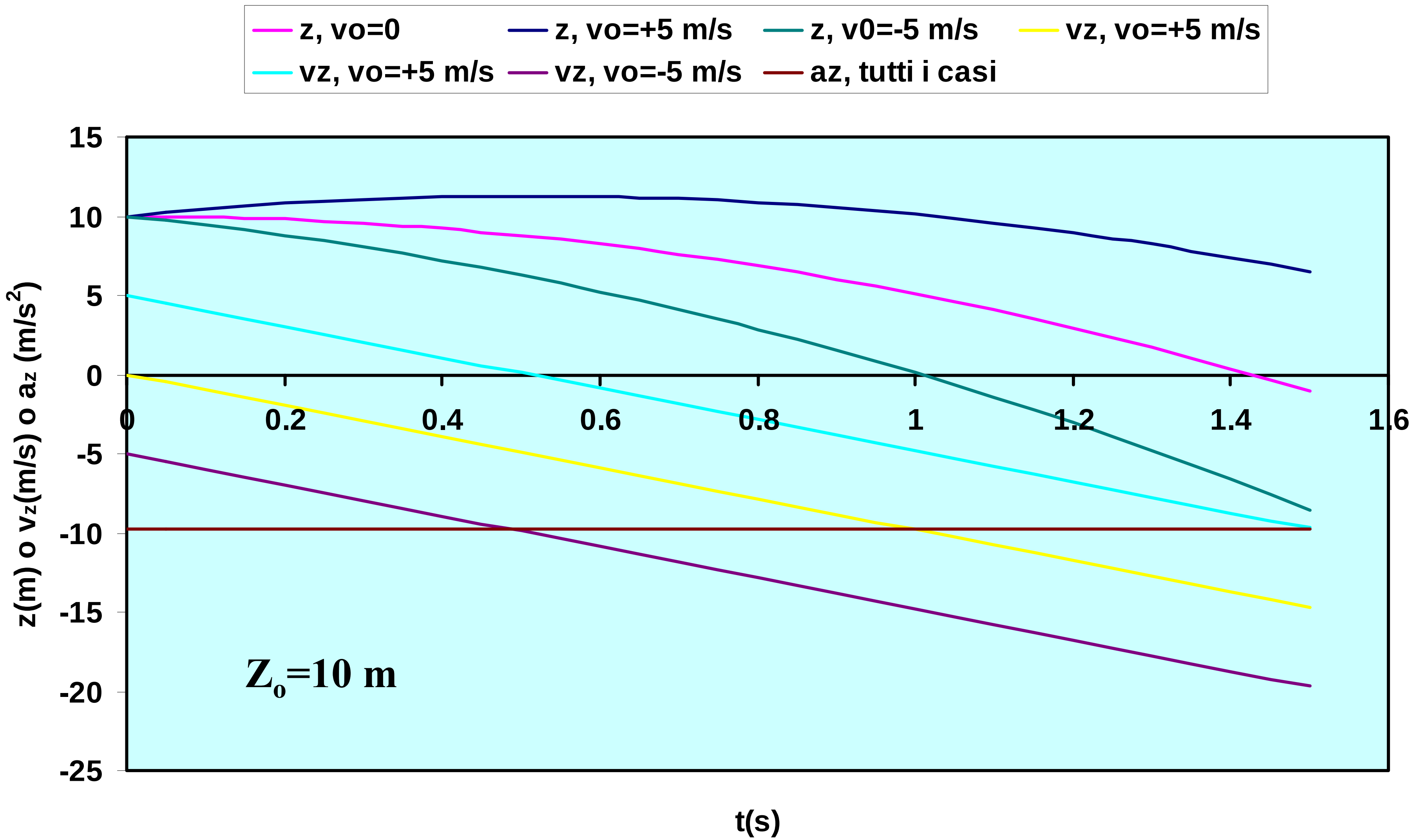
$$v_z(t) = v_o + a_o t$$

$$a_z(t) = a_o$$

$$a_o = -9.8\,\frac{m}{s^2}$$



$v_o = 0$ $\quad$ $v_o = +\,5\ m/s$ $\quad$ $v_o = -\,5\ m/s$


z, vo=0
z, vo=+5 m/s
z, v0=-5 m/s
vz, vo=+5 m/s
vz, vo=+5 m/s
vz, vo=-5 m/s
az, tutti i casi
15
10
5
0
-5
-10
-15
-20
-25
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
z(m) o vz(m/s) o az (m/s2)
t(s)
Zo=10 m

La velocità e l’accelerazione hanno versi indipendenti

La velocità può essere verso l’alto e l’accelerazione verso il basso o viceversa

2° caso: moto anche lungo x

$$z(t) = z_o + v_{zo}t + \frac{1}{2}a_o t^2$$

$$x(t) = x_o + v_{xo}t$$

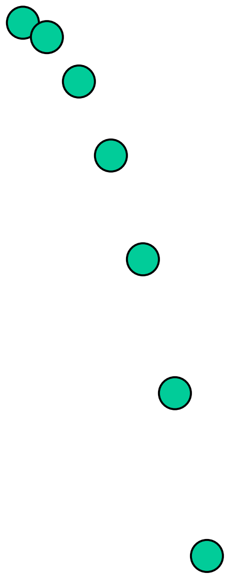
$$v_z(t) = v_{zo} + a_o t$$

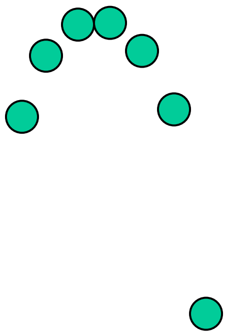
$$v_x(t) = v_{xo}$$

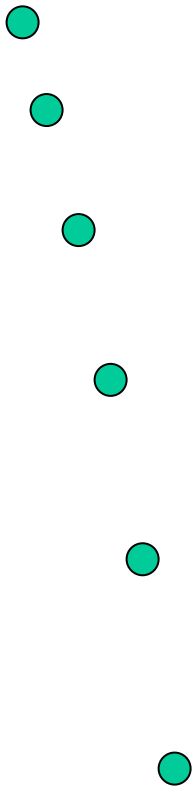
Composizione dei moti

v_{oz}=- 5 m/s

v_{ox}= 2m/s







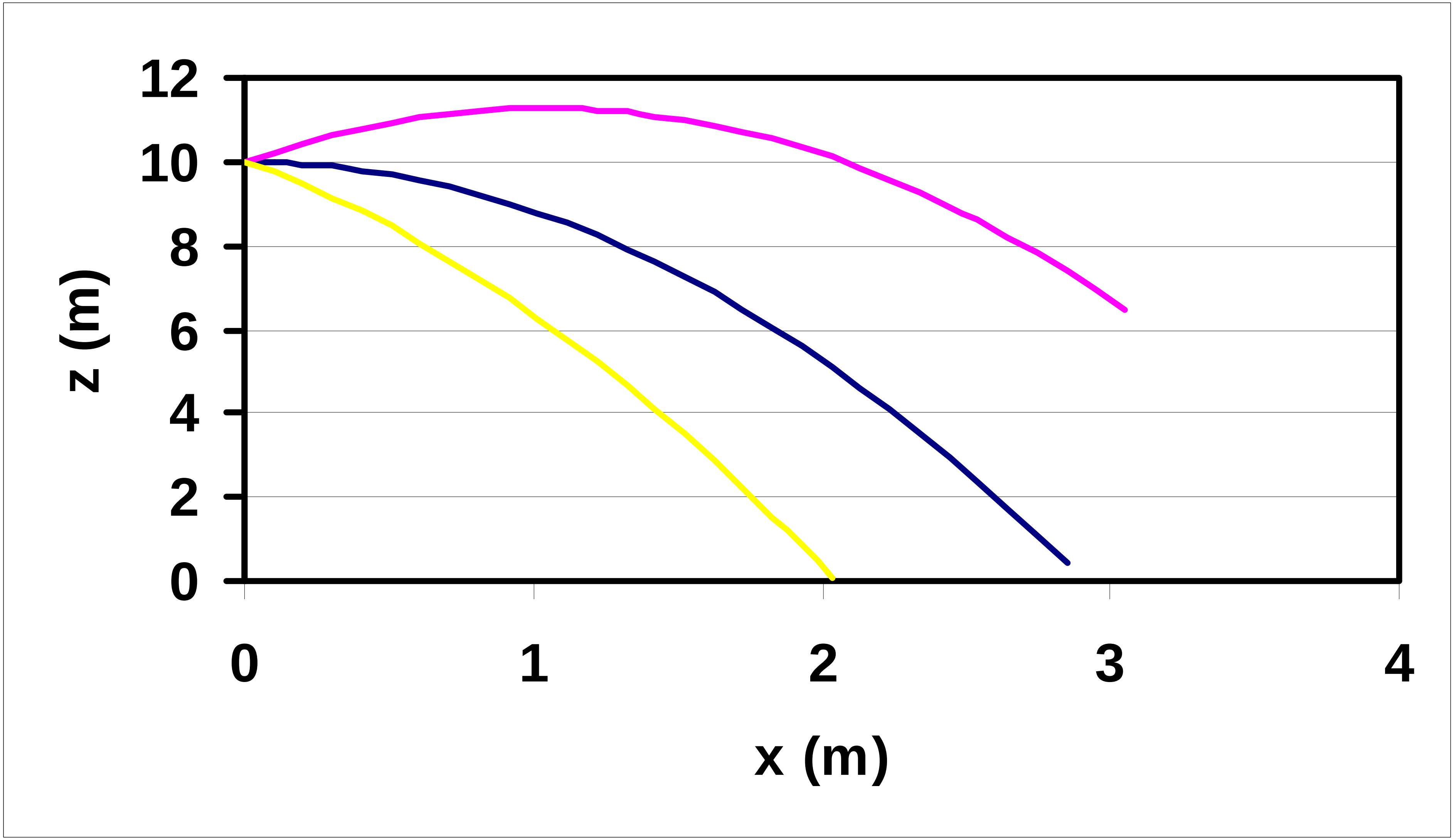
v_{oz}=0

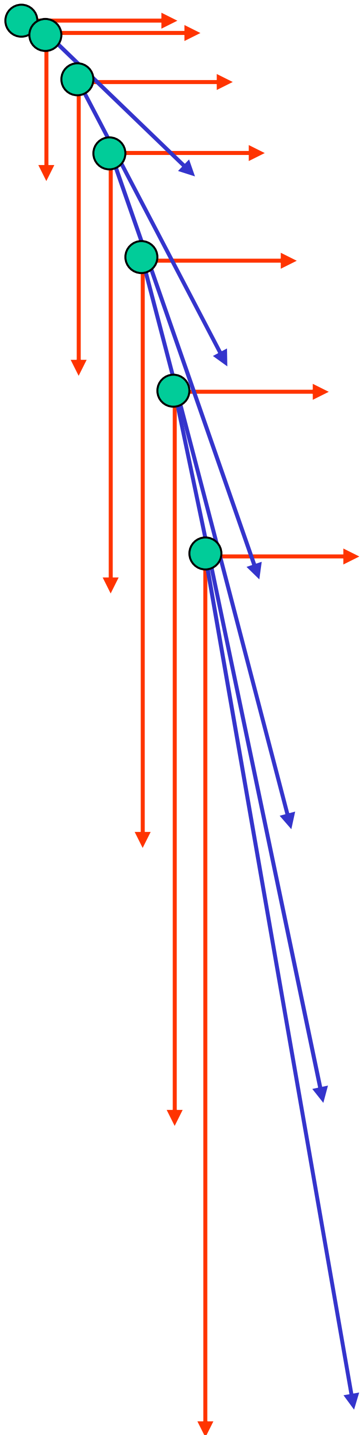
v_{ox}= 2m/s

v_{oz}=+ 5 m/s

v_{ox}= 2m/s

La traiettoria



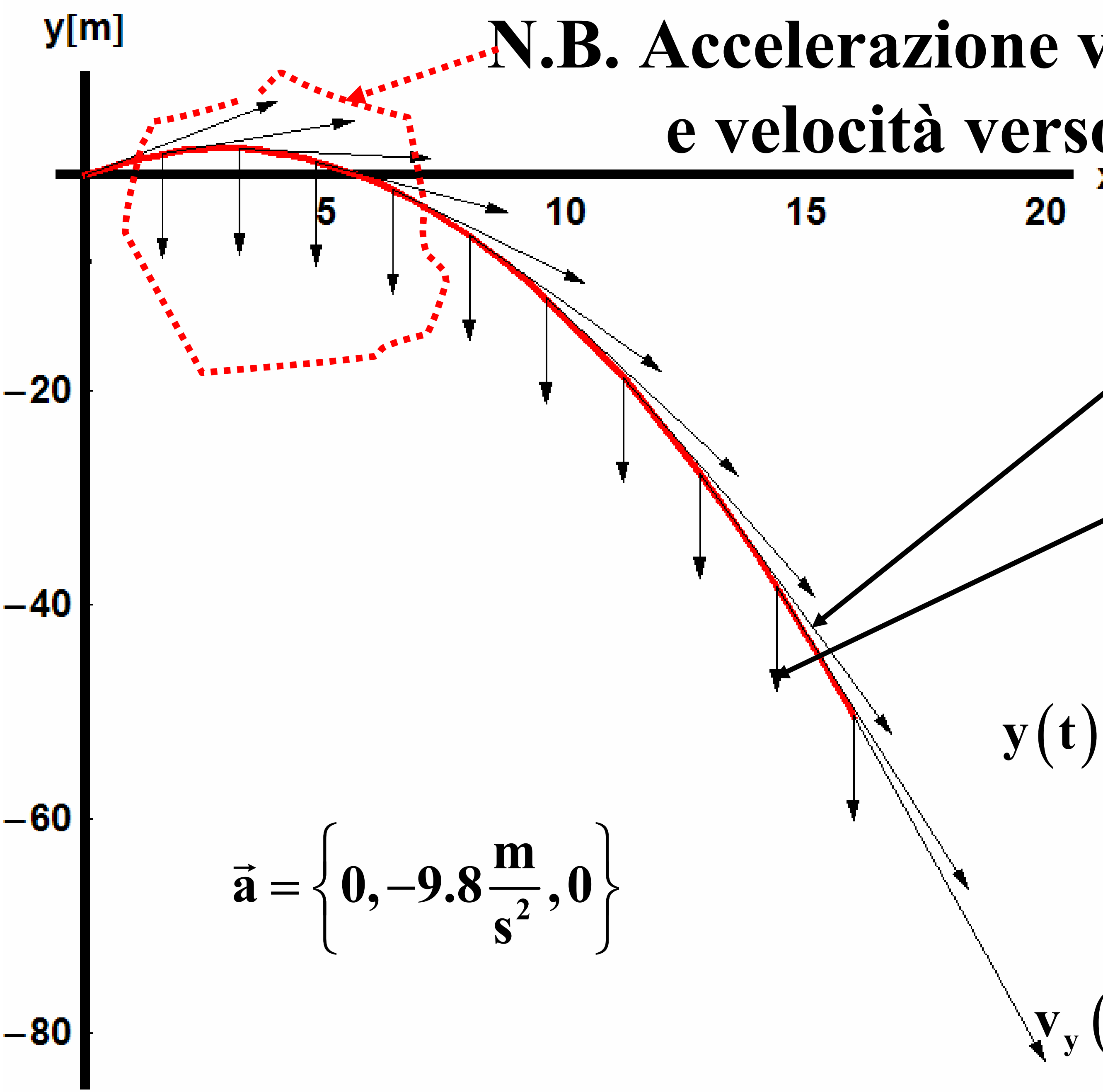


La composizione delle velocità

$$v_z(t) = -9.8\left(\frac{m}{s^2}\right)t$$

$$v_x(t) = 2\frac{m}{s}$$

N.B. Accelerazione verso il basso e velocità verso l'alto



$$\vec{a} = \left\{0, -9.8\frac{m}{s^2}, 0\right\}$$

$$x(t) = 4\frac{m}{s}t;$$

$$y(t) = 7\frac{m}{s}t - \frac{1}{2}9.8\frac{m}{s^2}t^2$$

$$v_x(t) = 4\frac{m}{s};$$

$$v_y(t) = 7\frac{m}{s} - 9.8\frac{m}{s^2}t$$

Un altro esempio

Un'esempio importante:

Il lancio di un proiettile: partenza dall'origine posta al suolo

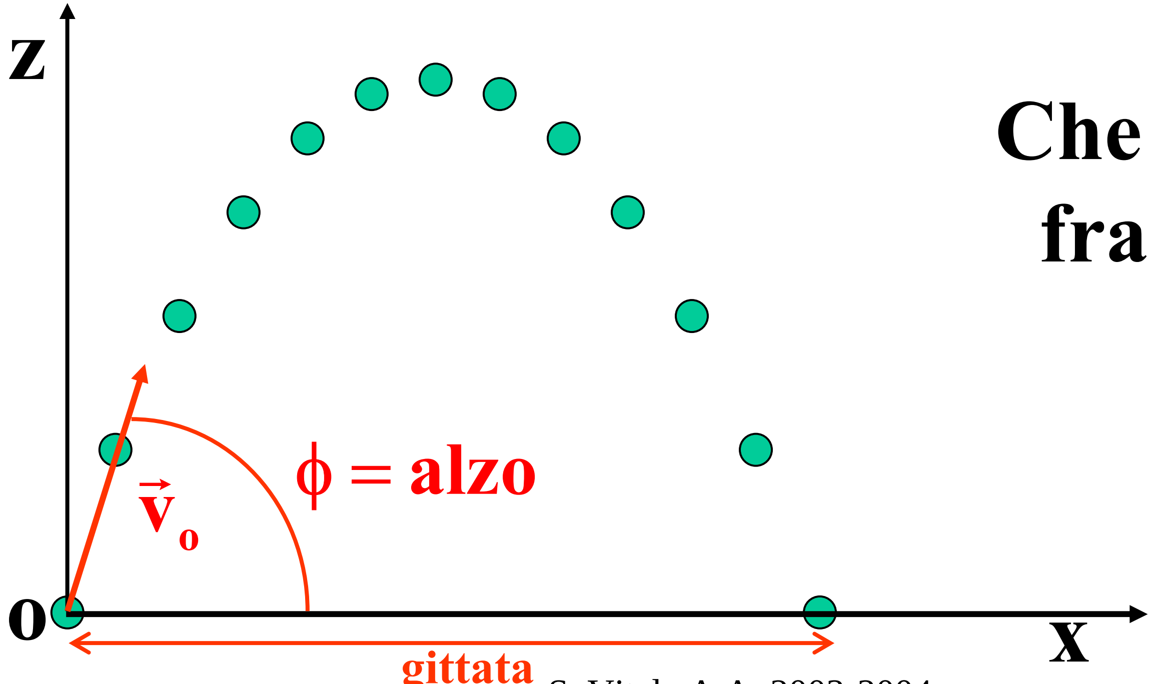
$$z(t) = v_{zo}t - \frac{1}{2}gt^2$$

$$x(t) = v_{xo}t$$

$$g = 9.8\frac{m}{s^2}$$

$$v_z(t) = v_{zo} - gt$$

$$v_x(t) = v_{xo}$$

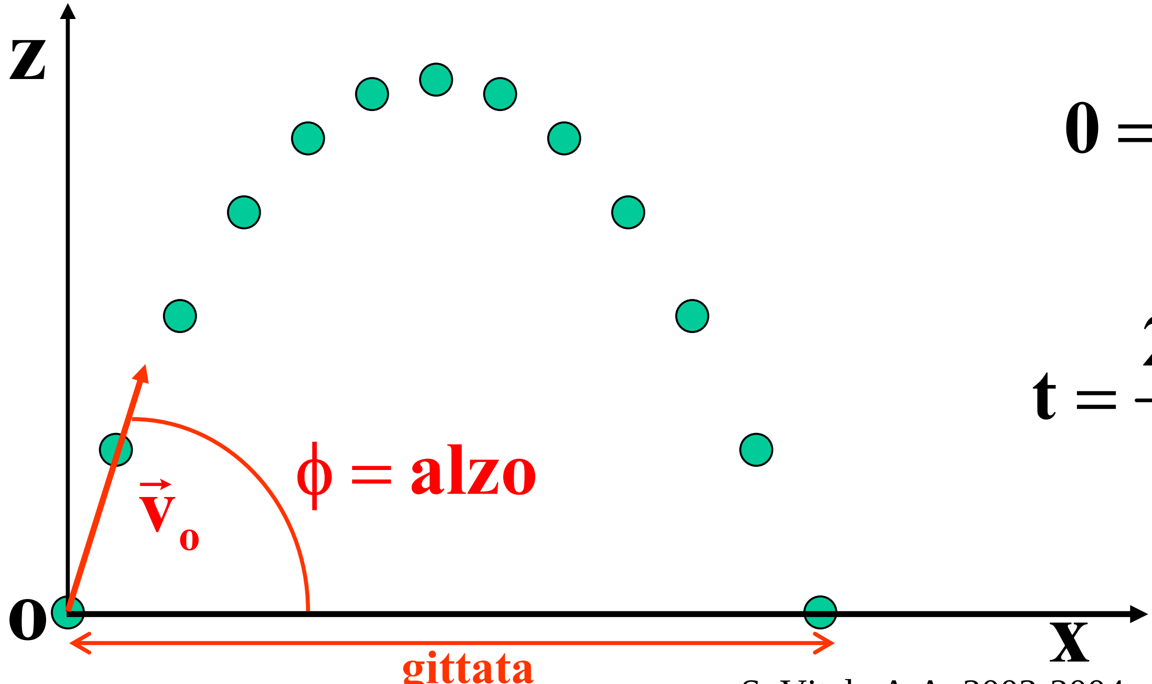


Che relazione c'è fra alzo, v_o e la gittata?

$$v_{ox} = |\vec{v}_o| \mathrm{Cos}(\phi) \qquad v_{oz} = |\vec{v}_o| \mathrm{Sin}(\phi)$$

$$z(t) = |\vec{v}_o| \mathrm{Sin}(\phi) t - \frac{1}{2} g t^2 \qquad x(t) = |\vec{v}_o| \mathrm{Cos}(\phi) t$$

$$v_z(t) = |\vec{v}_o| \mathrm{Sin}(\phi) - g t \qquad v_x(t) = |\vec{v}_o| \mathrm{Cos}(\phi)$$



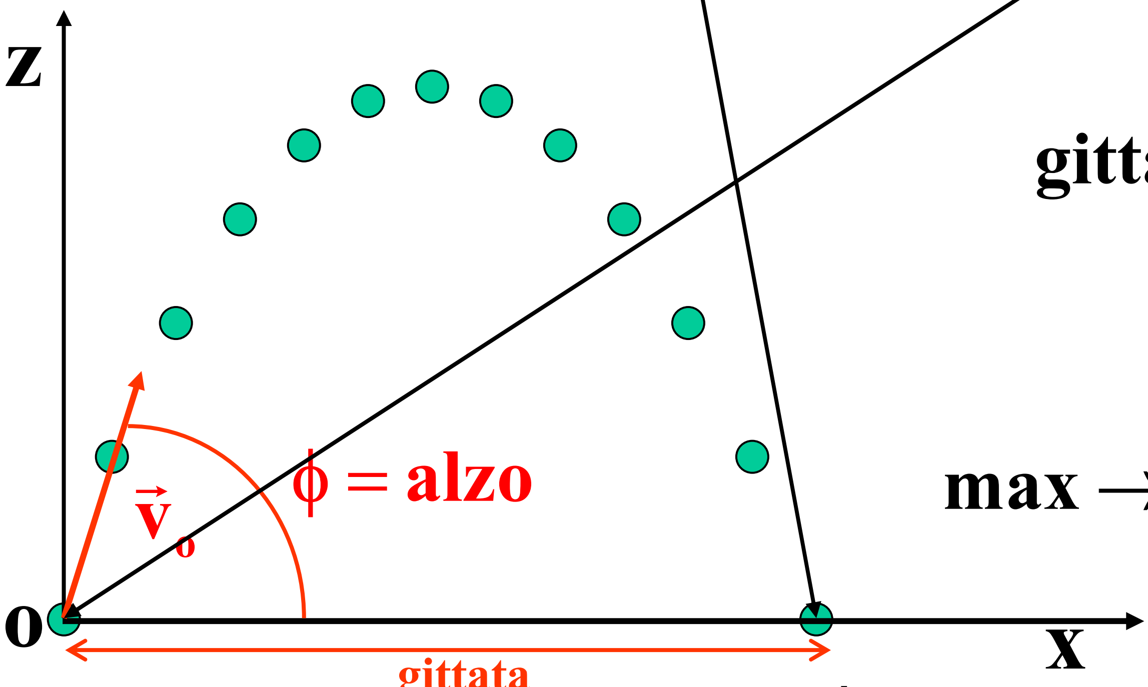
Impatto: z(t)=0

$$0 = \left[|\vec{v}_o| \mathrm{Sin}(\phi) - \frac{1}{2} g t \right] t$$

$$t = \frac{2 |\vec{v}_o| \mathrm{Sin}(\phi)}{g} \leftrightarrow t = 0$$

$$x(t) = |\vec{v}_o| \mathrm{Cos}(\phi)\, t \qquad t = \frac{2|\vec{v}_o| \mathrm{Sin}(\phi)}{g} \leftrightarrow t = 0$$

$$x = \frac{2|\vec{v}_o|^2 \mathrm{Sin}(\phi)\mathrm{Cos}(\phi)}{g} \leftrightarrow x = 0$$



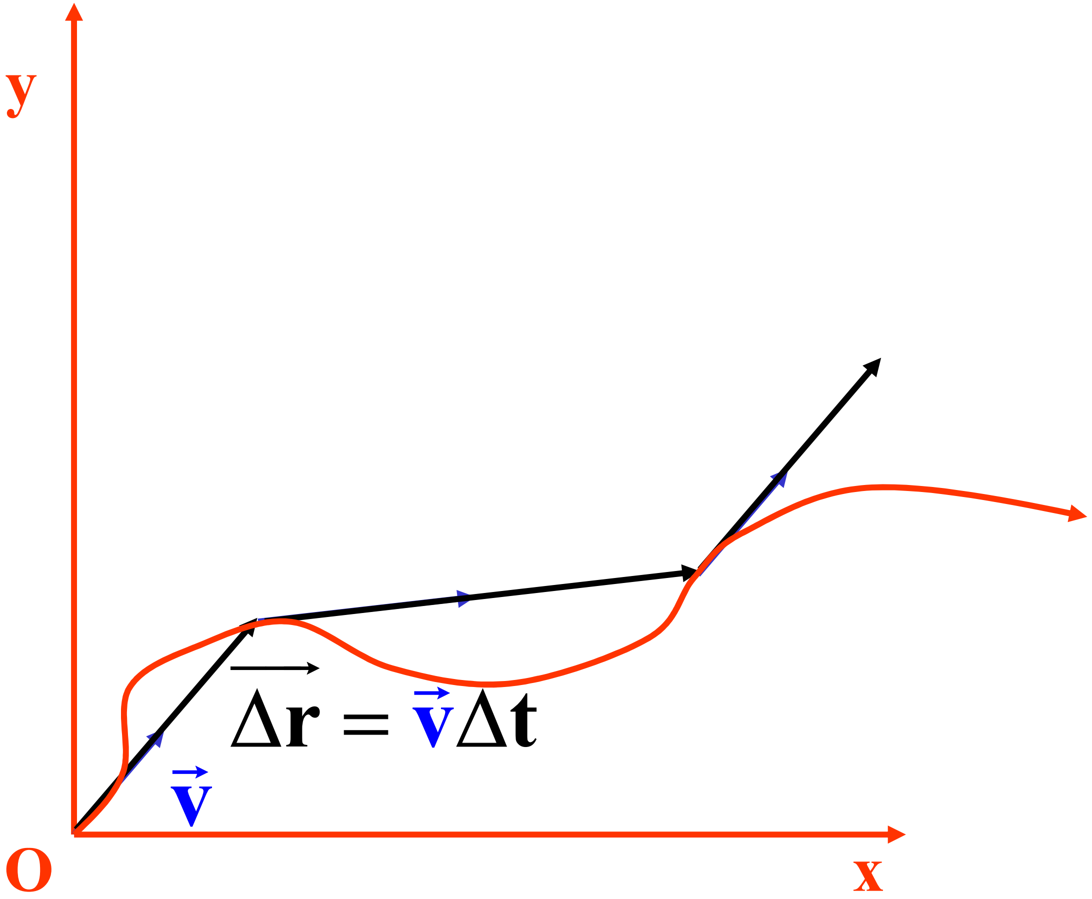
$$\text{gittata} = \frac{|\vec{v}_o|^2 \mathrm{Sin}(2\phi)}{g}$$

$$\max \rightarrow \frac{|\vec{v}_o|^2 \mathrm{Sin}\left(2\frac{\pi}{4}\right)}{g} = \frac{|\vec{v}_o|^2}{g}$$

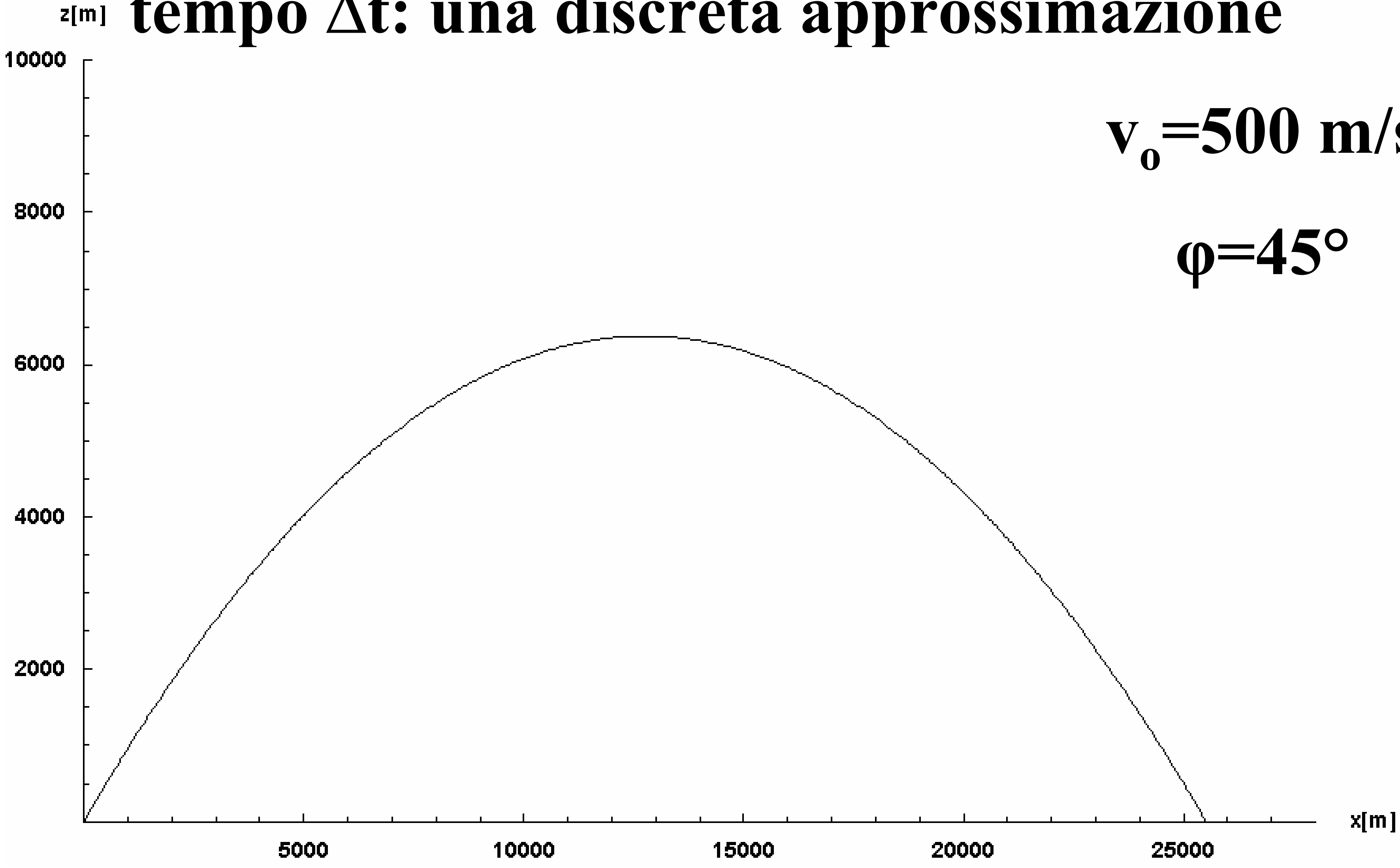
Perchè l'accelerazione è importante?

$$\vec{F} = m\vec{a}$$

Ricostruzione della legge oraria dalla velocità



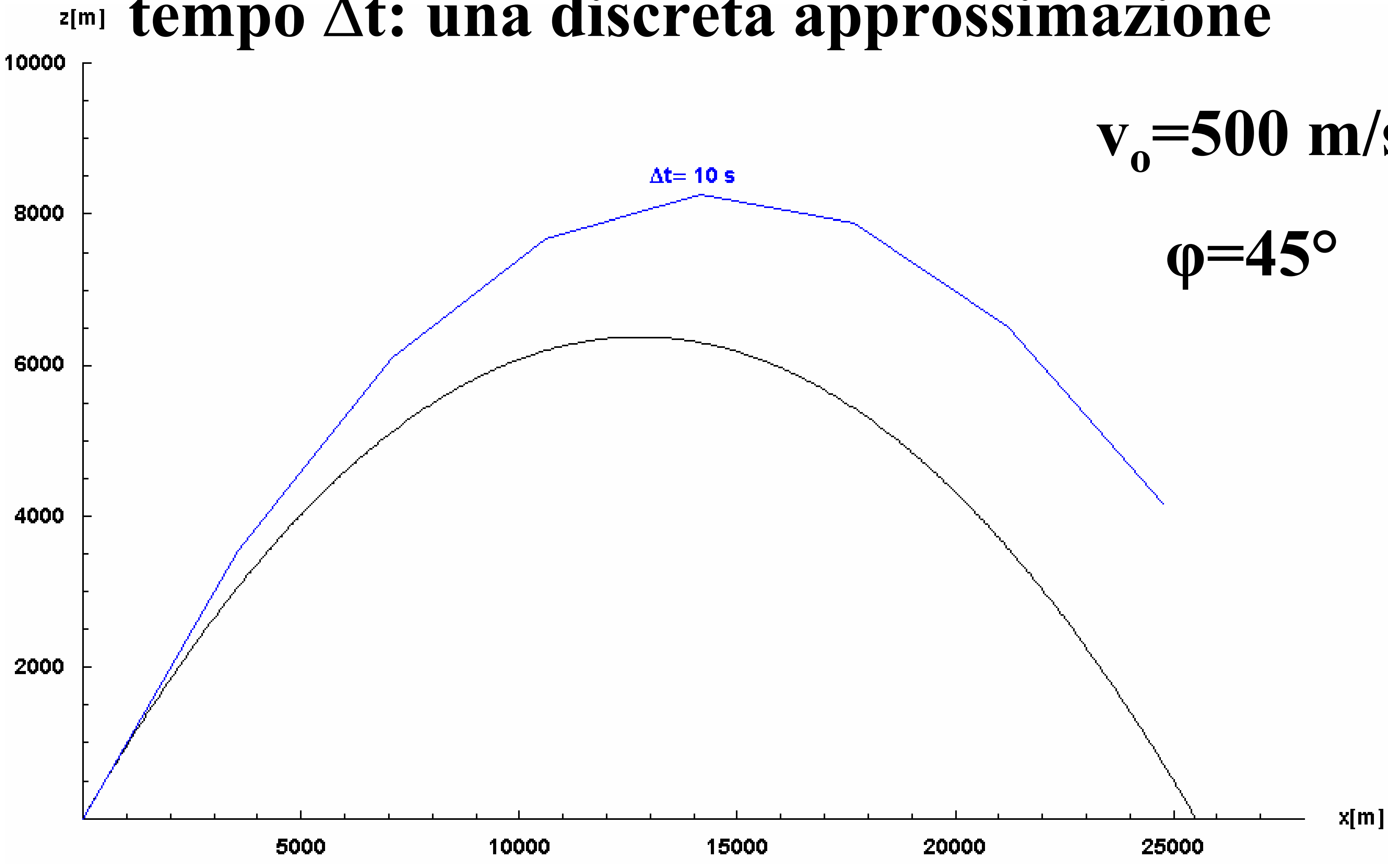
Andare per la tangente a velocità costante per un tempo Δt: una discreta approssimazione



v_o=500 m/s

φ=45°

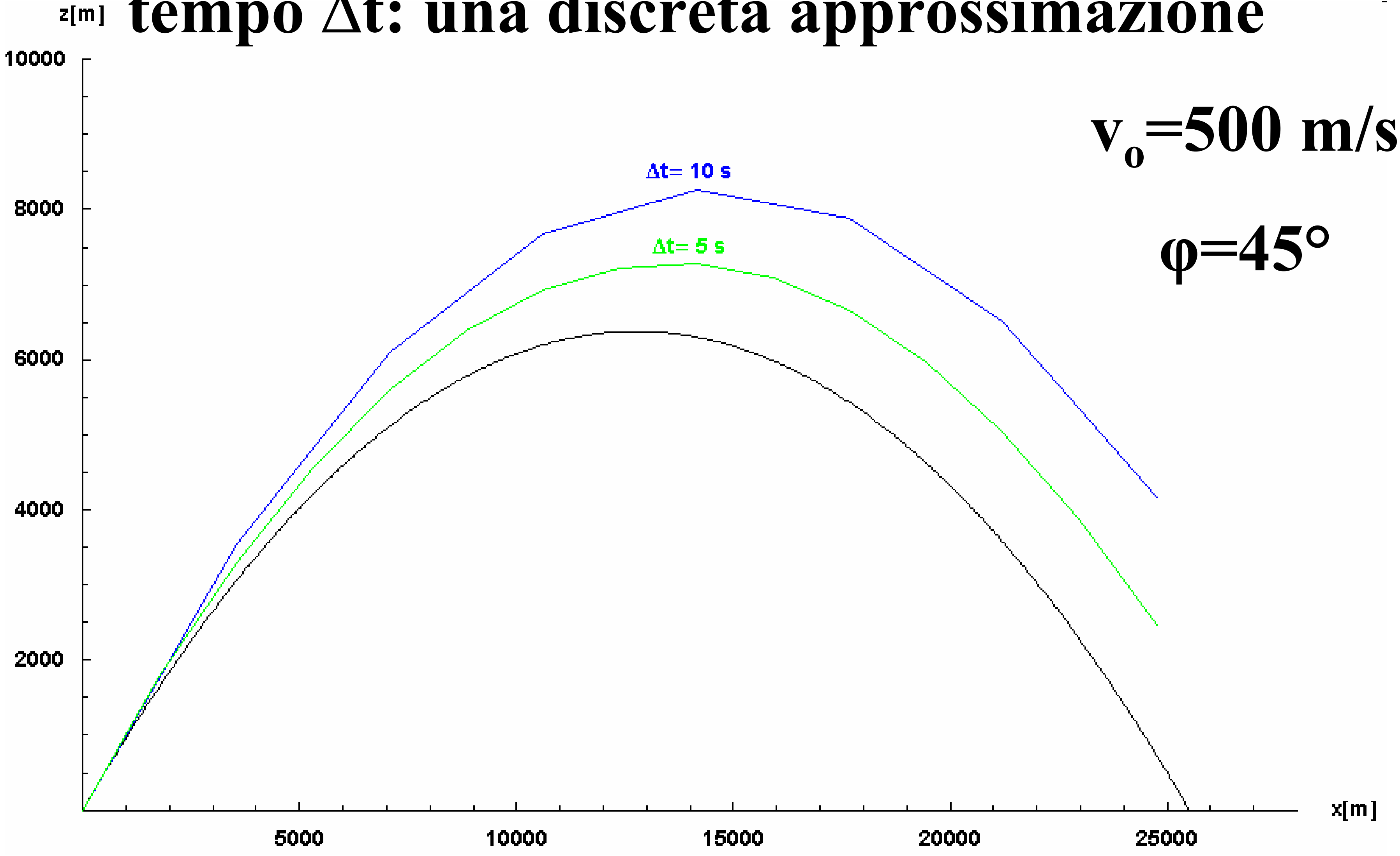
Andare per la tangente a velocità costante per un tempo Δt: una discreta approssimazione



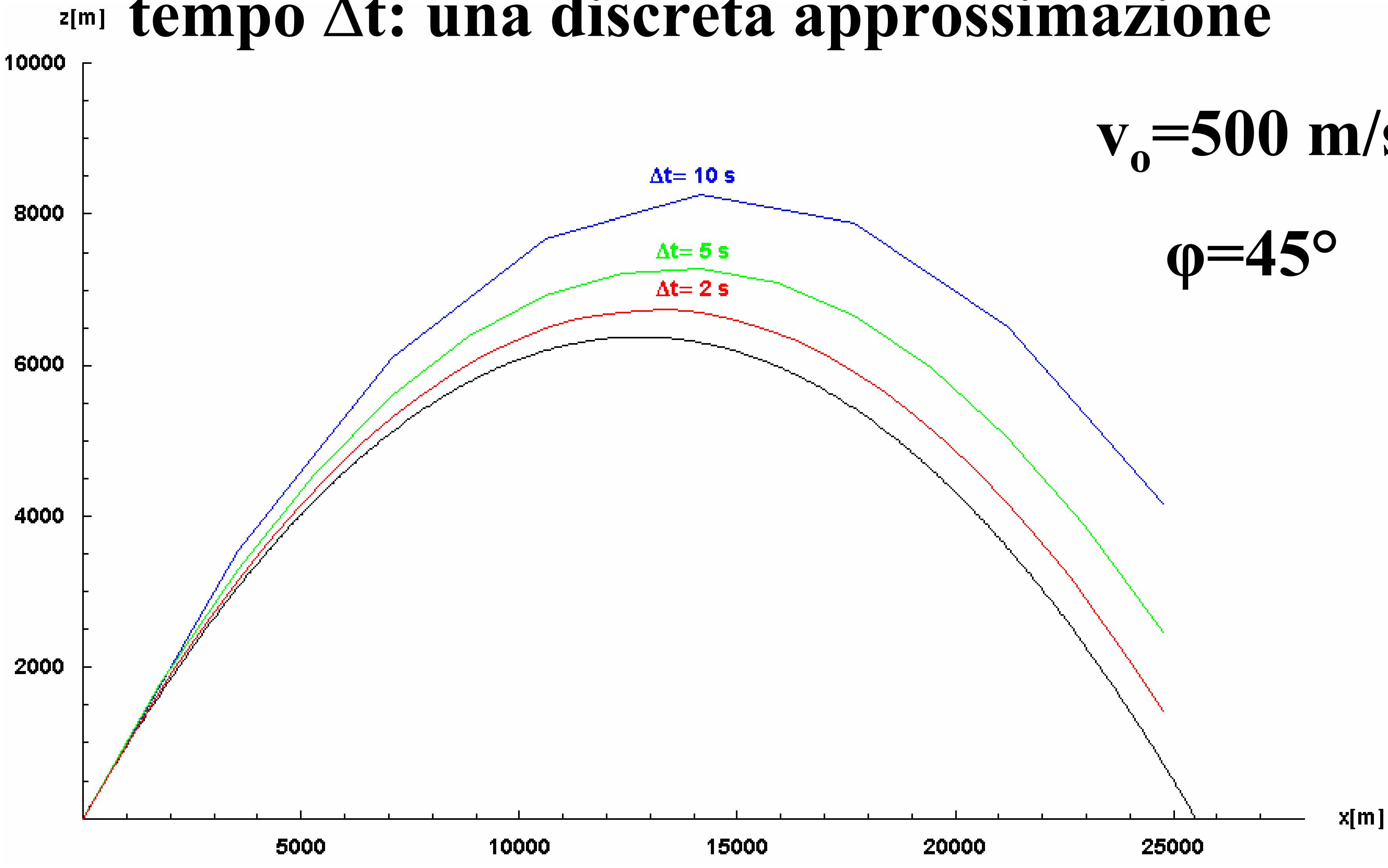
v_o=500 m/s

φ=45°

Andare per la tangente a velocità costante per un tempo Δt: una discreta approssimazione



Andare per la tangente a velocità costante per un tempo Δt: una discreta approssimazione



v_o=500 m/s

φ=45°

Ricostruzione della legge oraria dalla velocità

$$v_x(t) = \frac{dx}{dt} \quad \rightarrow \quad x(t_B) - x(t_A) = \int_{t_A}^{t_B} v_x(t)\,dt$$

$$v_y(t) = \frac{dy}{dt} \quad \rightarrow \quad y(t_B) - y(t_A) = \int_{t_A}^{t_B} v_y(t)\,dt$$

$$v_z(t) = \frac{dz}{dt} \quad \rightarrow \quad z(t_B) - z(t_A) = \int_{t_A}^{t_B} v_z(t)\,dt$$

$$\vec{v}(t) = \frac{d\vec{r}}{dt} \quad \rightarrow \quad \vec{r}(t_B) - \vec{r}(t_A) = \int_{t_A}^{t_B} \vec{v}(t)\,dt$$

$$\vec{r}(t_B) = \vec{r}(t_A) + \int_{t_A}^{t_B} \vec{v}(t)\,dt$$

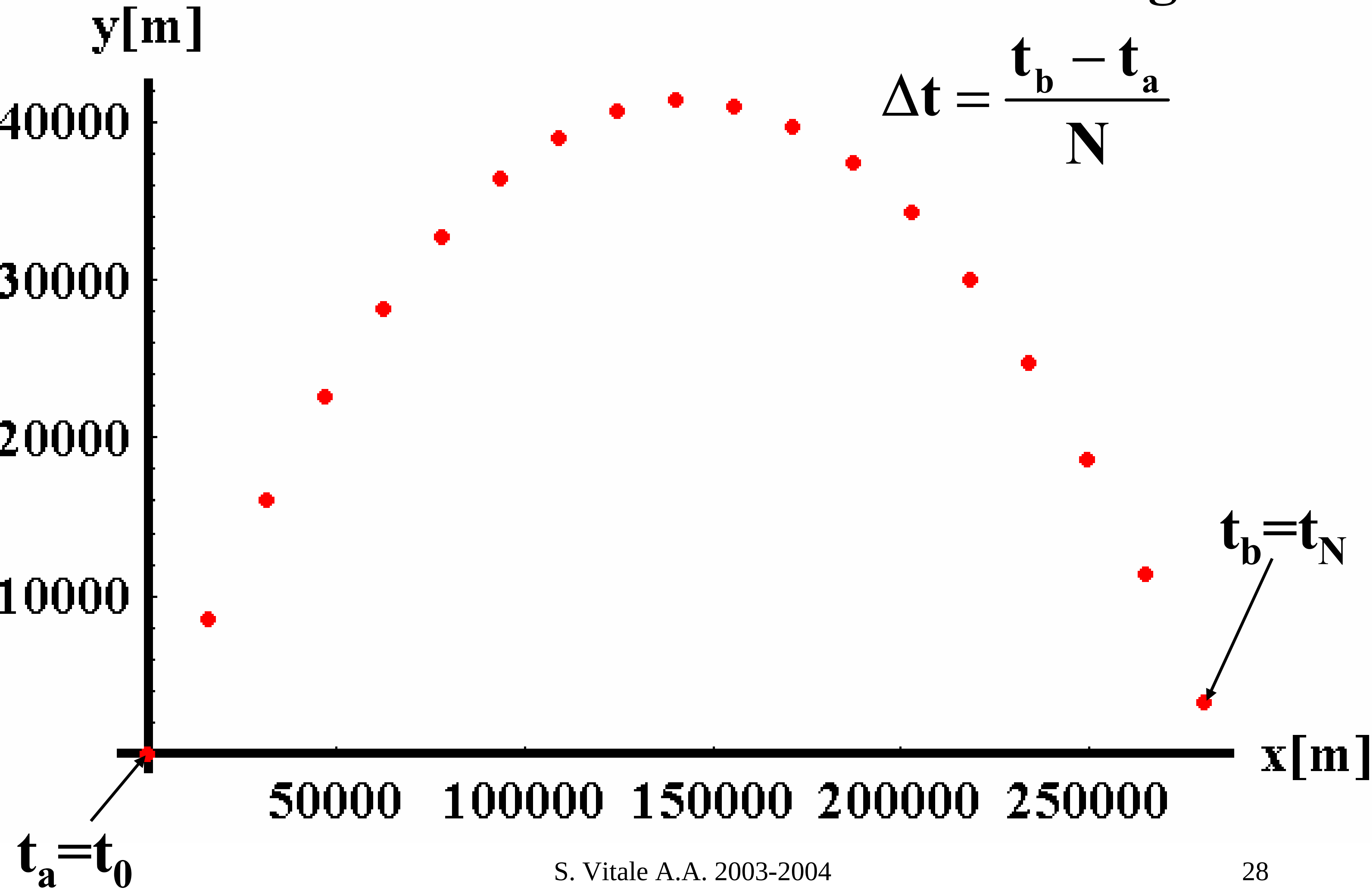
Per conoscere la posizione al tempo t_B bisogna conoscere la velocità fra t_A e t_B

e la posizione al tempo t_A

Perché l'integrale?

Dividiamo $[t_a,t_b]$ in N intervalli lunghi

$$\Delta t = \frac{t_b - t_a}{N}$$



$$x[t_B]-x[t_A]\equiv x[t_N]-x[t_0]=$$

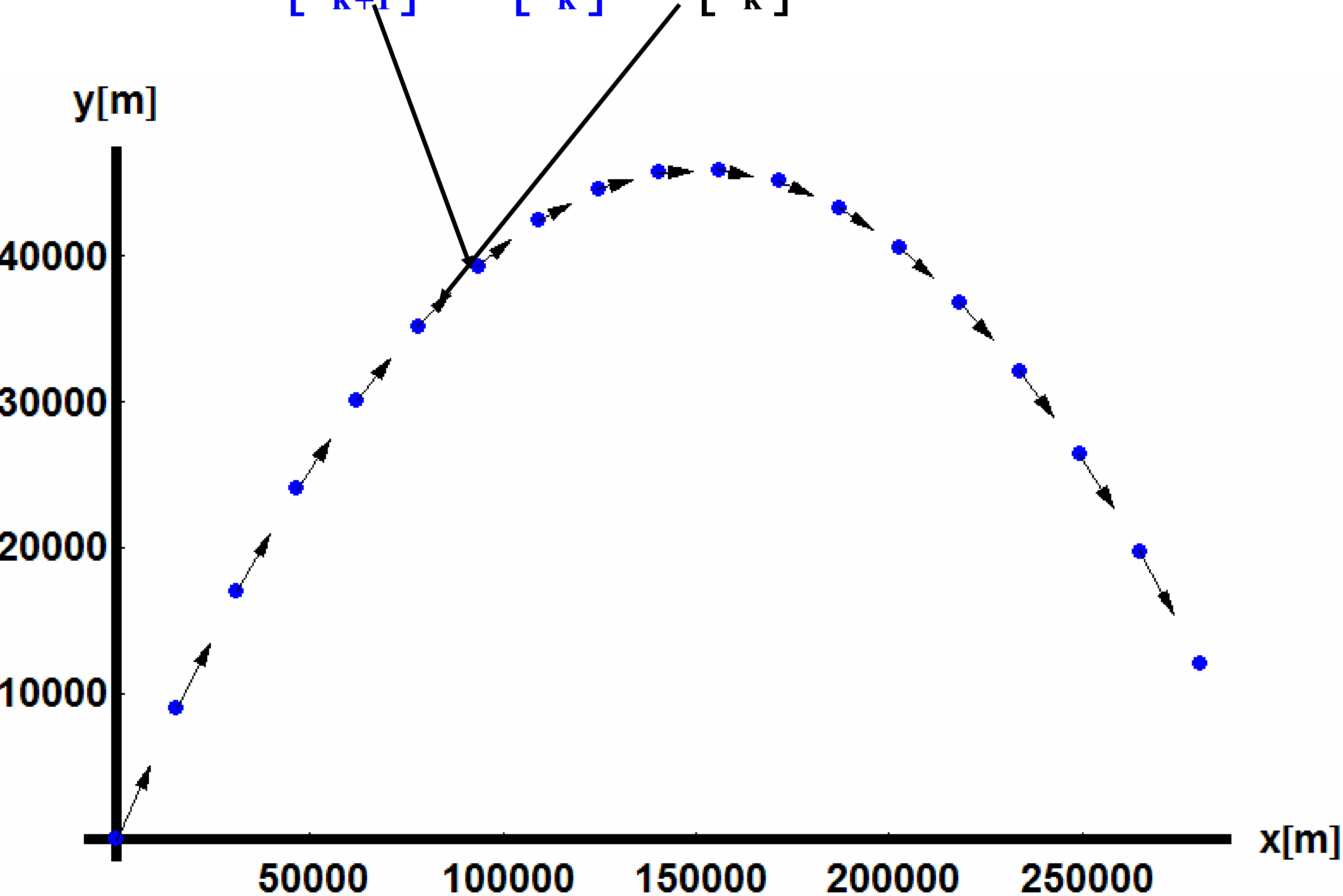
$$x[t_1]-x[t_0]+x[t_2]-x[t_1]+x[t_3]-x[t_2]+\ldots+x[t_N]$$

$$=\sum_{k=0}^{N-1}\left(x[t_{k+1}]-x[t_k]\right)$$

$$\frac{x[t_{k+1}=t_k+\Delta t]-x[t_k]}{\Delta t}\approx v_x[t_k]$$

$$\sum_{k=0}^{N-1}\left(x[t_{k+1}]-x[t_k]\right)\approx\sum_{k=0}^{N-1}v_x[t_k]\Delta t$$

$$\vec{r}[t_{k+1}] \approx \vec{r}[t_k] + \vec{v}[t_k]\Delta t$$



$$\sum_{k=0}^{N-1}\left(x\left[t_{k+1}\right]-x\left[t_k\right]\right)\approx\sum_{k=0}^{N-1}v_x\left[t_k\right]\Delta t$$

$$x\left[t_A\right]-x\left[t_B\right]=\underset{\substack{\Delta t\to 0\\ \left(N=\frac{t_B-t_A}{\Delta t}\to\infty\right)}}{\mathrm{Lim}}\sum_{k=0}^{N-1}v_x\left[t_k\right]\Delta t\equiv\int_{t_A}^{t_B}v_x\left[t\right]dt$$

Es: moto rettilineo uniforme

$$v_x(t) = v_{xo};\; v_y(t) = v_{yo}; v_z(t) = v_{zo};$$

$$\vec{v}(t) = \{v_x(t),\; v_y(t), v_z(t)\} = \{v_{xo}, v_{yo}, v_{zo}\} \equiv \vec{v}_o$$

$$x(t_2) = x(t_1) + \int_{t_1}^{t_2} v_{xo}\,dt = x(t_1) + v_{xo}(t_2 - t_1)$$

$$y(t_2) = y(t_1) + \int_{t_1}^{t_2} v_{yo}\,dt = y(t_1) + v_{yo}(t_2 - t_1)$$

$$z(t_2) = z(t_1) + \int_{t_1}^{t_2} v_{zo}\,dt = z(t_1) + v_{zo}(t_2 - t_1)$$

$$\vec{r}(t_2) = \vec{x}(t_1) + \int_{t_1}^{t_2} \vec{v}_o\,dt = \vec{r}(t_1) + \vec{v}_o(t_2 - t_1)$$

N.B. da

$$\vec{r}(t_B) = \vec{r}(t_A) + \int_{t_A}^{t_B} \vec{v}(t)\,dt$$

Segue che la velocità media

$$\bar{\vec{v}}(t_A, t_B) \equiv \frac{\vec{r}(t_B) - \vec{r}(t_A)}{t_B - t_A} = \frac{1}{t_B - t_A} \int_{t_A}^{t_B} \vec{v}(t)\,dt$$

È la media temporale della velocità istantanea

Come si ricavano posizione e velocità dall'accelerazione?

$$\vec{v}(t_2) - \vec{v}(t_1) = \int_{t_1}^{t_2} \vec{a}(t)\,dt$$

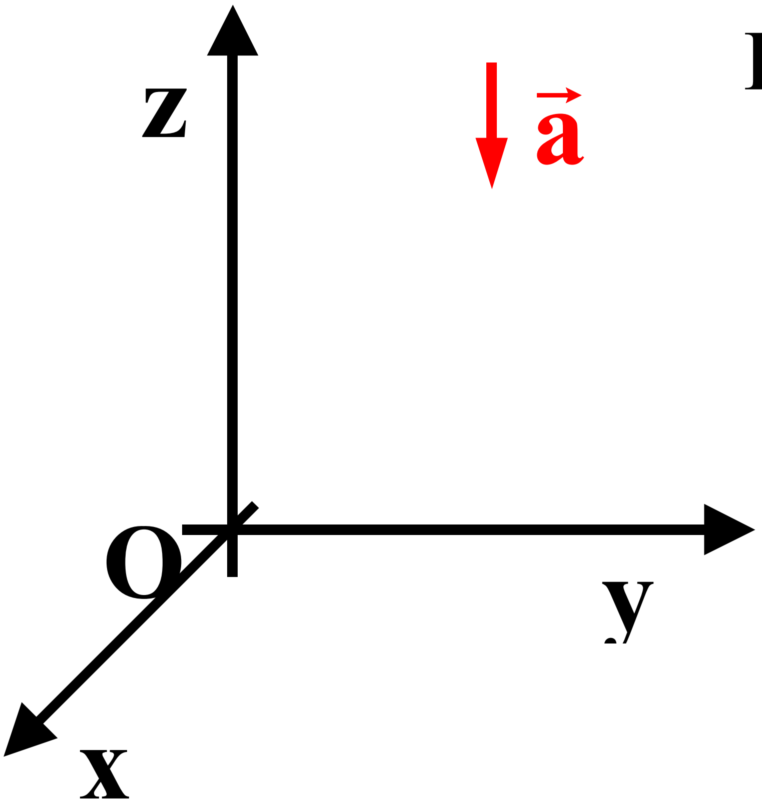
$$\vec{v}(t) = \vec{v}(t_o) + \int_{t_o}^{t} \vec{a}(t')\,dt'$$

$$\vec{r}(t) = \vec{r}(t_o) + \int_{t_o}^{t} \vec{v}(t')\,dt' = \vec{r}(t_o) + \int_{t_o}^{t} \left[\vec{v}(t_o) + \int_{t_o}^{t'} \vec{a}(t'')\,dt'' \right] dt'$$

$$\vec{r}(t) = \vec{r}(t_o) + \vec{v}(t_o)(t - t_o) + \int_{t_o}^{t} dt' \int_{t_o}^{t'} \vec{a}(t'')\,dt''$$

Due condizioni iniziali: se l'accelerazione è nulla la velocità può essere diversa da zero

Esempio: il moto nel campo gravitazionale terrestre



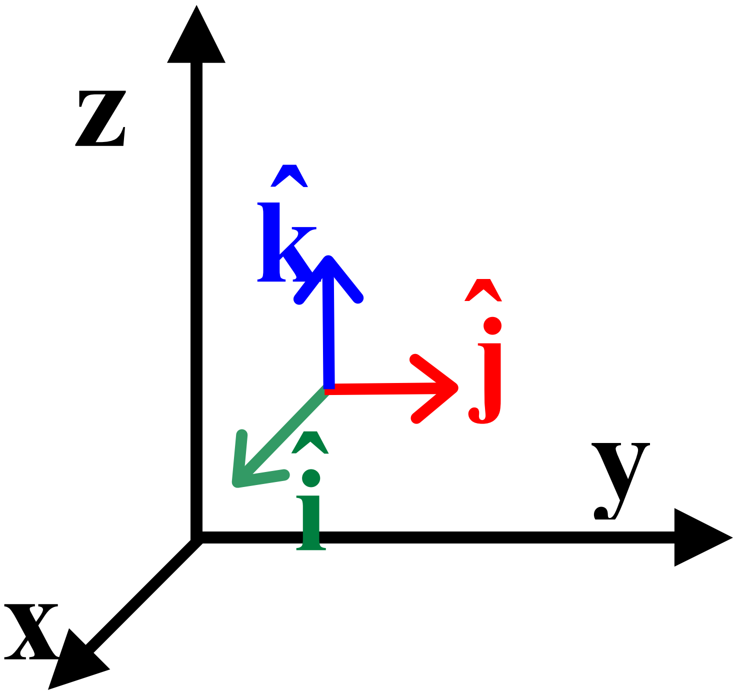
$$\vec{a}(t) = -g\hat{k}$$

$$g=9.8\ m/s^2$$

Una piccola parentesi matematica: il versore

$$\hat{B} \equiv \frac{\vec{B}}{|\vec{B}|} = \left\{ \frac{B_x}{\sqrt{B_x^2 + B_y^2 + B_z^2}}, \frac{B_y}{\sqrt{B_x^2 + B_y^2 + B_z^2}}, \frac{B_z}{\sqrt{B_x^2 + B_y^2 + B_z^2}} \right\}$$

$$\rightarrow |\hat{B}| = 1 \rightarrow \vec{B} = |\vec{B}|\hat{B}$$



$$\hat{i} = \{1,0,0\}$$

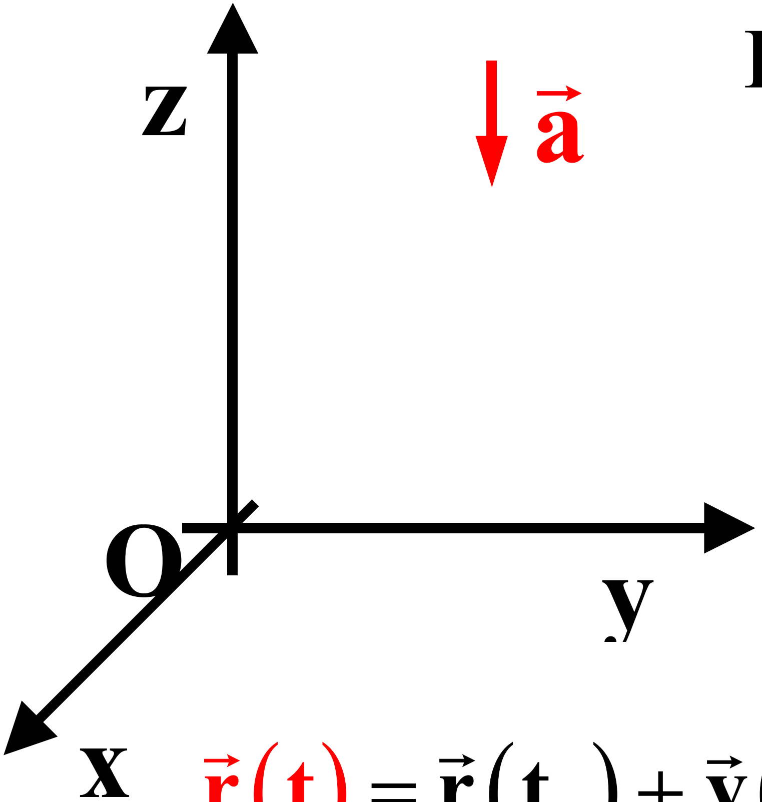
$$\hat{j} = \{0,1,0\}$$

$$\hat{k} = \{0,0,1\}$$

$$\vec{B} = \{B_x, B_y, B_z\} = \{B_x, 0, 0\} + \{0, B_y, 0\} + \{0, 0, B_z\} =$$

$$B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

Esempio: il moto nel campo gravitazionale terrestre



$$\vec{a}(t) = -g\hat{k}$$

$g=9.8\ m/s^2$

$$\vec{r}(t) = \vec{r}(t_o) + \vec{v}(t_o)(t - t_o) + \int_{t_o}^{t} dt' \int_{t_o}^{t'} -g\hat{k}\ dt'' =$$

$$= \vec{r}(t_o) + \vec{v}(t_o)(t - t_o) - \int_{t_o}^{t} g\hat{k}(t' - t_o) dt' =$$

$$= \vec{r}(t_o) + \vec{v}(t_o)(t - t_o) - \hat{k}\ \frac{1}{2} g (t - t_o)^2$$

La velocità

$$\vec{v}(t) = \vec{v}(t_o) - g\hat{k}(t - t_o)$$

$$v_x(t) = v_x(t_o);\ v_y(t) = v_y(t_o)$$

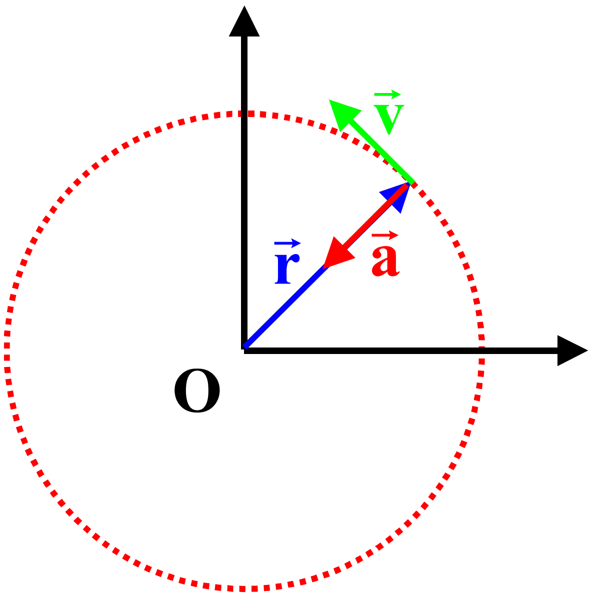
$$v_z(t) = v_z(t_o) - g(t - t_o)$$

Proprietà dell'accelerazione

1: Moto circolare uniforme

$$\vec{v}(t) = \{-\omega\, r_o \mathrm{Sin}(\omega t),\ \omega\, r_o \mathrm{Cos}(\omega t),\ 0\} \quad |\vec{v}(t)| = \omega r_o$$

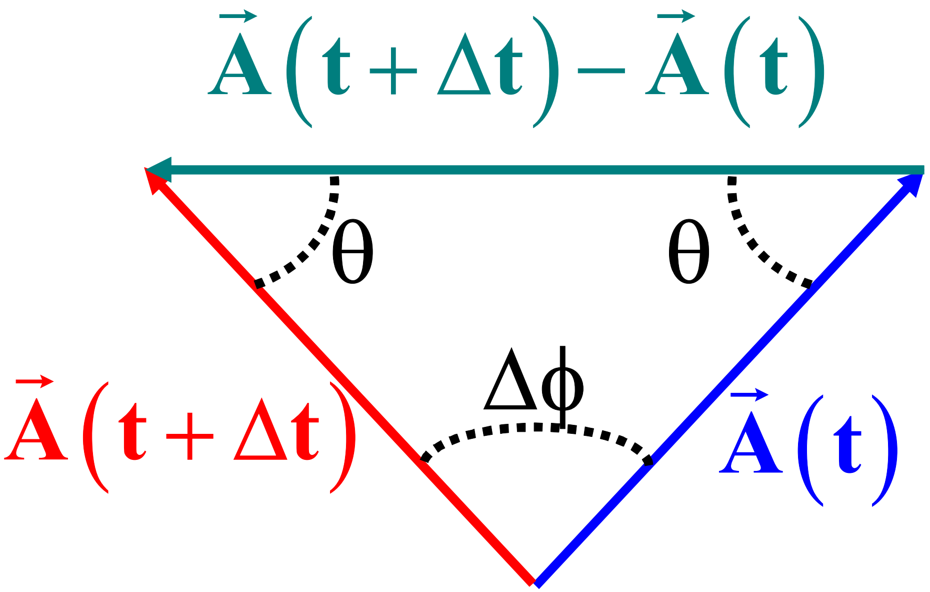
$$\vec{a}(t) = \frac{d\vec{v}(t)}{dt} = \{-\omega^2\, r_o \mathrm{Cos}(\omega t),\ -\omega^2\, r_o \mathrm{Sin}(\omega t),\ 0\} = -\omega^2 \vec{r}(t)$$



$$\vec{v} \cdot \vec{a} = 0$$

$$|\vec{a}(t)| = \omega^2 r_o = \frac{|\vec{v}|^2}{r_o}$$

Derivata di un vettore costante in modulo



$$\Delta\phi = \pi - 2\theta; \ \theta = \frac{\pi}{2} - \frac{\Delta\phi}{2}$$

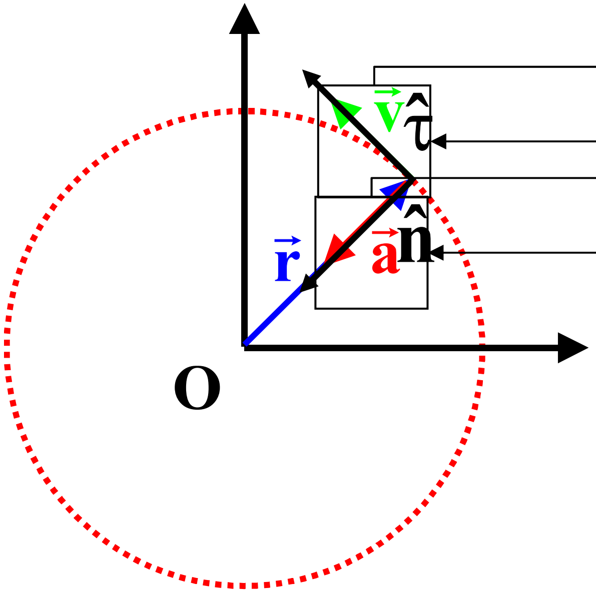
$$\Delta\phi \to 0 \quad \Rightarrow \theta \to \frac{\pi}{2}$$

$$\left|\frac{\vec{A}(t+\Delta t)-\vec{A}(t)}{\Delta t}\right| = \frac{2}{\Delta t}\left|\vec{A}(t)\right|\left|\mathrm{Sin}\left(\frac{\Delta\phi}{2}\right)\right| \approx \frac{|\Delta\phi|}{\Delta t}\left|\vec{A}(t)\right|$$

$$\lim_{\Delta t\to 0}\left|\frac{\vec{A}(t+\Delta t)-\vec{A}(t)}{\Delta t}\right| = \left|\frac{d\phi}{dt}\right|\left|\vec{A}(t)\right|$$

$$|\vec{v}| = \omega_o |\vec{r}| = \omega_o r_o$$

$$|\vec{a}| = \omega_o |\vec{v}| = \omega_o^2 |\vec{r}| = \omega_o^2 r_o$$

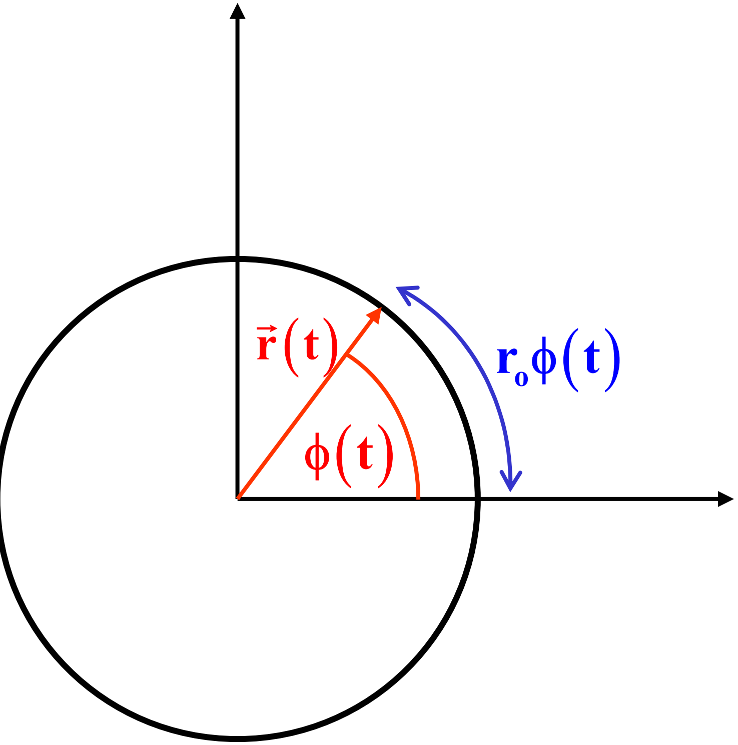


$\vec{v} = \omega_o r_o \hat{\tau}$ $\quad \hat{\tau} \equiv$ versore tangente

$\vec{a} = \omega_o^2 r_o \hat{n}$ $\quad \hat{n} \equiv$ versore normale

Es: Moto circolare non uniforme

$$\vec{r}(t)=\{r_o\mathrm{Cos}[\phi(t)],r_o\mathrm{Sin}[\phi(t)],0\}\quad |\vec{r}(t)|=r_o$$



$$\vec{v}(t)=\left\{-r_o\mathrm{Sin}[\phi(t)]\frac{d\phi(t)}{dt},\right.$$

$$\left.r_o\mathrm{Cos}[\phi(t)]\frac{d\phi(t)}{dt},0\right\}$$

$$|\vec{v}(t)|=r_o\left|\frac{d\phi(t)}{dt}\right|$$

$$\vec{v}(t)=\left\{-r_o \mathrm{Sin}[\phi(t)]\frac{d\phi(t)}{dt}, r_o \mathrm{Cos}[\phi(t)]\frac{d\phi(t)}{dt}, 0\right\}$$

$$\left|\vec{v}(t)\right| = r_o\left|\frac{d\phi(t)}{dt}\right|$$

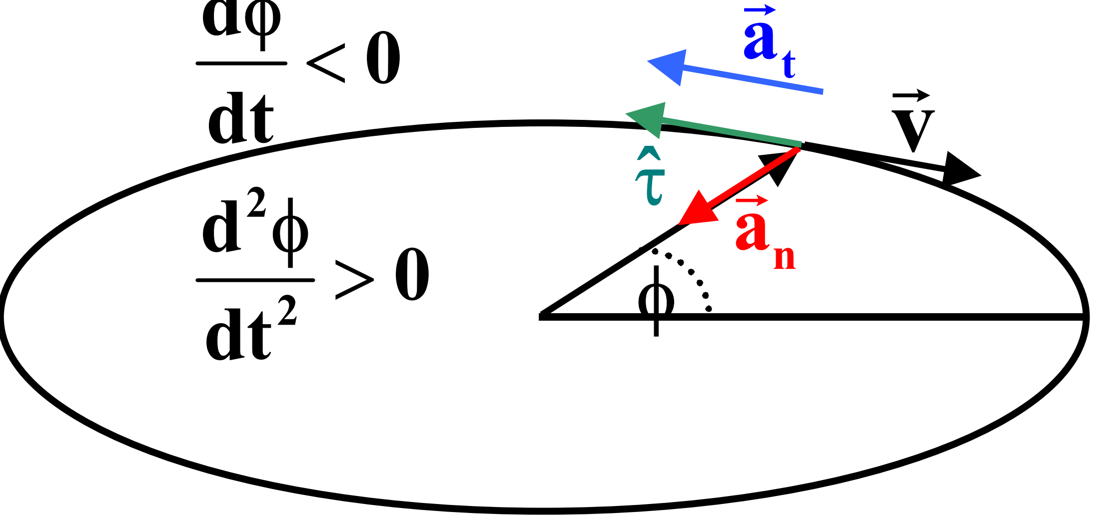
$$\vec{a}(t)=\left\{-r_o \mathrm{Cos}[\phi(t)]\left(\frac{d\phi(t)}{dt}\right)^2 - r_o \mathrm{Sin}[\phi(t)]\frac{d^2\phi(t)}{dt^2},\right.$$

$$\left.-r_o \mathrm{Sin}[\phi(t)]\left(\frac{d\phi(t)}{dt}\right)^2 + r_o \mathrm{Cos}[\phi(t)]\frac{d^2\phi(t)}{dt^2}, 0\right\} =$$

$$= -r_o\left(\frac{d\phi(t)}{dt}\right)^2 \hat{r}(t) + r_o\frac{d^2\phi(t)}{dt^2}\hat{\tau}(t)$$

Centripeta

Tangenziale



$$\frac{d\phi}{dt} < 0$$

$$\frac{d^2\phi}{dt^2} > 0$$

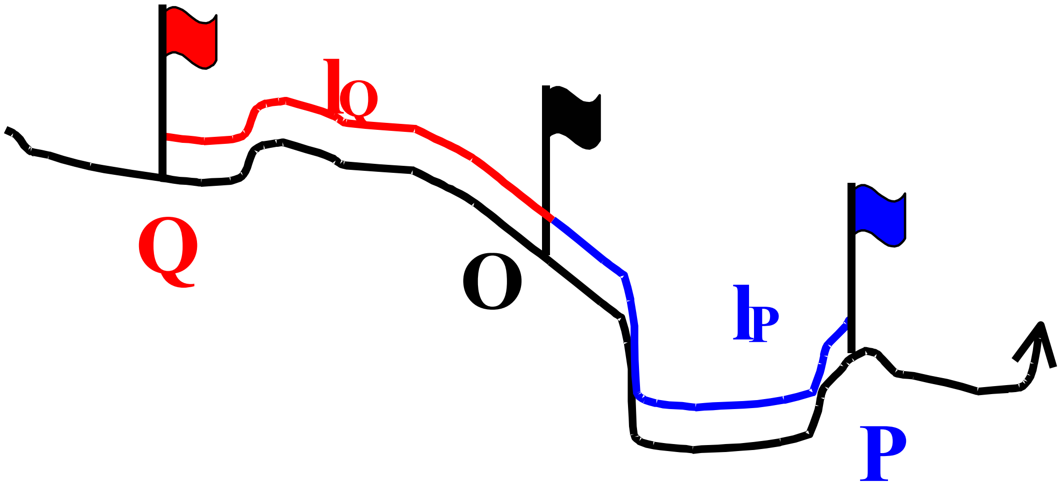
$$\vec{a}(t) = \vec{a}_n + \vec{a}_t$$

$$\vec{v}(t) = r_o \frac{d\phi(t)}{dt}\hat{\tau}$$

$$\vec{a}_n(t) = -r_o\left(\frac{d\phi(t)}{dt}\right)^2 \hat{r}(t)$$

$$\left|\vec{a}_n\right| = \frac{\left|\vec{v}(t)\right|^2}{r_o}$$

$$\vec{a}_t = r_o \frac{d^2\phi(t)}{dt^2}\hat{\tau}(t)$$



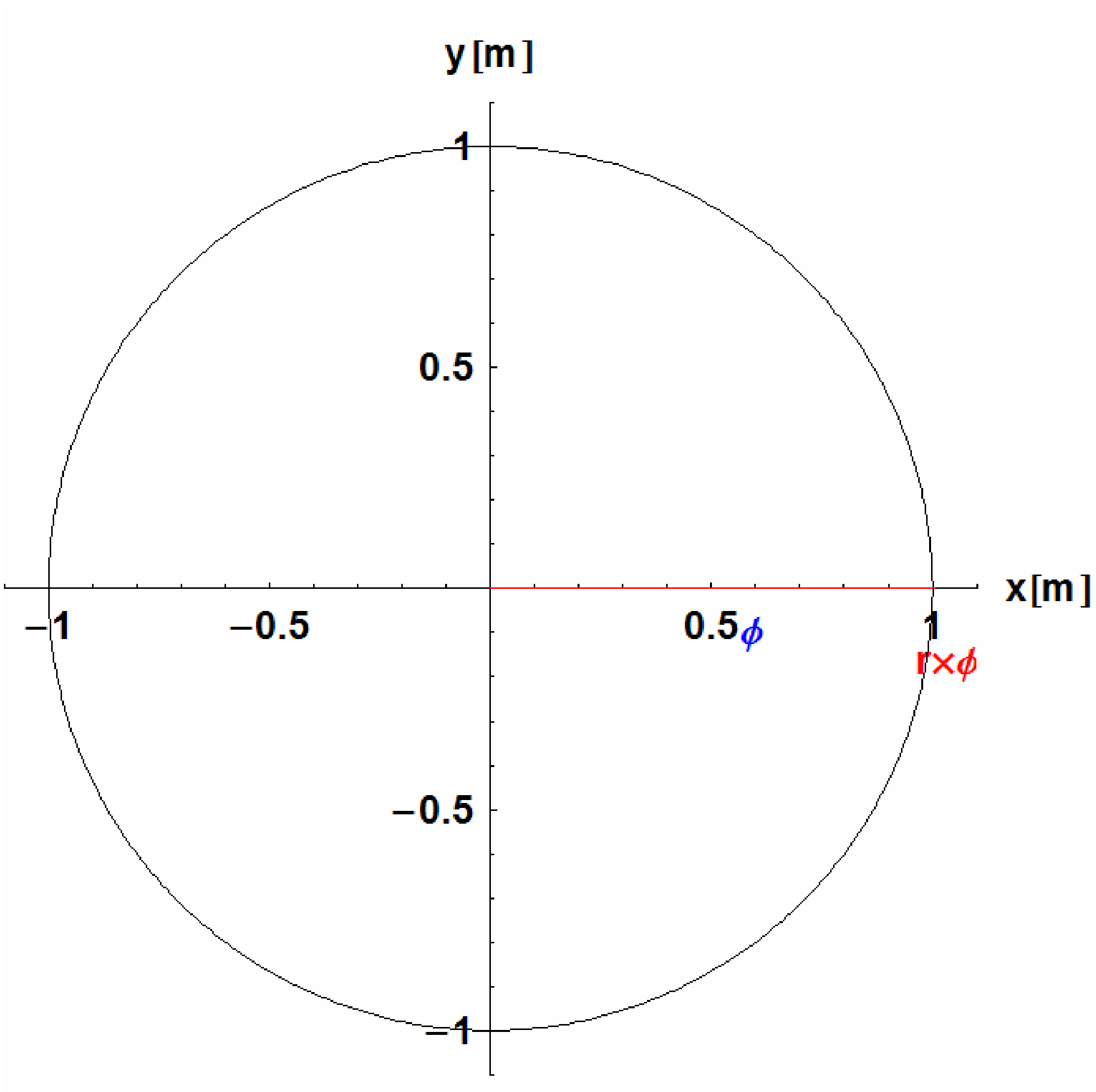
$$s_P = l_P$$
$$s_Q = -l_Q$$

Ascissa Curvilinea s su una curva orientata: Distanza di un punto sulla curva da un'origine sulla stessa

+ : il punto segue l'origine
-: il punto precede l'origine

$$|\vec{v}| = \frac{d(\text{lunghezza dell'arco di traiettoria percorso})}{d(\text{tempo impiegato a percorrerlo})} = \left|\frac{ds}{dt}\right|$$

Moto Circolare



Moto Circolare

$$v_x(t) = -\frac{d\phi(t)}{dt}\, r_o \mathrm{Sin}\left[\phi(t)\right]; v_y(t) = \frac{d\phi(t)}{dt}\, r_o \mathrm{Cos}\left[\phi(t)\right];$$

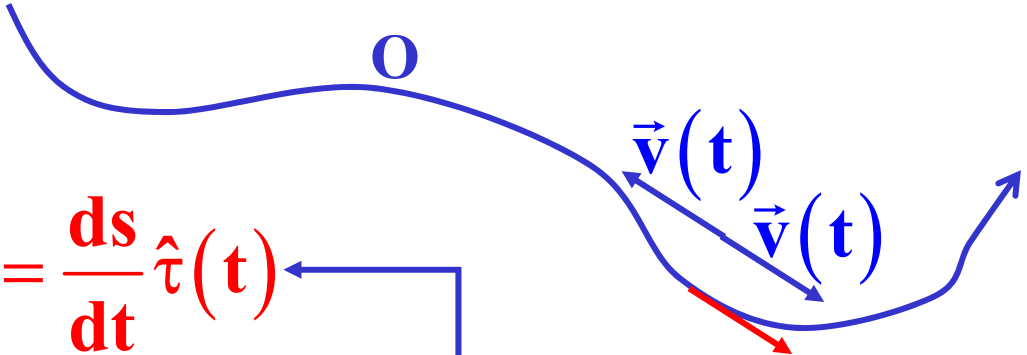
$$v_z(t) = 0$$

$$\sqrt{v_x^2(t) + v_y^2(t) + v_z^2(t)} =$$

$$\sqrt{\left(\frac{d\phi(t)}{dt}\, r_o\right)^2 \mathrm{Sin}^2\left[\phi(t)\right] + \left(\frac{d\phi(t)}{dt}\, r_o\right)^2 \mathrm{Cos}^2\left[\phi(t)\right]}$$

$$= r_o \left|\frac{d\phi(t)}{dt}\right| = \left|\frac{ds(t)}{dt}\right|$$

$$s(t) = s(0) + \int_0^t \frac{d\phi(t)}{dt} r_o dt = s(0) + r_o\left[\phi(t) - \phi(0)\right]$$



$$\vec{v}(t) = \frac{ds}{dt}\hat{\tau}(t)$$

$\hat{\tau} \equiv$ **versore tangente**

Il verso è quello positivo della traiettoria

$$\left|\vec{v}(t)\right| = \left|\frac{ds}{dt}\right|\left|\hat{\tau}\right| = \left|\frac{ds}{dt}\right|$$

$$\frac{ds}{dt} \geq 0 \rightarrow \vec{v} \uparrow\uparrow \hat{\tau}$$

$$\frac{ds}{dt} \leq 0 \rightarrow \vec{v} \uparrow\downarrow \hat{\tau}$$

$$\vec{v}(t) = \left|\vec{v}(t)\right|\hat{v}(t)$$

Derivata del prodotto di uno scalare per un vettore

$$\frac{d a(t)\vec{A}(t)}{dt} = \frac{d\left\{a(t)A_x(t), a(t)A_y(t), a(t)A_z(t)\right\}}{dt}$$

$$= \left\{\begin{array}{l} \frac{da(t)}{dt}A_x(t) + a(t)\frac{dA_x(t)}{dt}, \\ \frac{da(t)}{dt}A_y(t) + a(t)\frac{dA_y(t)}{dt}, \frac{da(t)}{dt}A_z(t) + a(t)\frac{dA_z(t)}{dt} \end{array}\right\}$$

$$= \frac{da(t)}{dt}\vec{A}(t) + a(t)\frac{d\vec{A}(t)}{dt}$$

Applichiamolo alla velocità

$$\frac{d\vec{v}(t)}{dt} = \frac{d\left[|v|\hat{v}\right]}{dt} = \frac{d|v|}{dt}\hat{v} + |v|\frac{d\hat{v}}{dt} = \pm\frac{d^2s}{dt^2}\hat{\tau} + |\vec{v}|\frac{d\hat{v}}{dt}$$

$\vec{a}_t$

Accelerazione tangenziale

$$|\vec{a}_t| = \left|\frac{d^2s}{dt^2}\right|$$

$$\frac{d\hat{v}(t)}{dt} \perp \hat{v}(t)$$

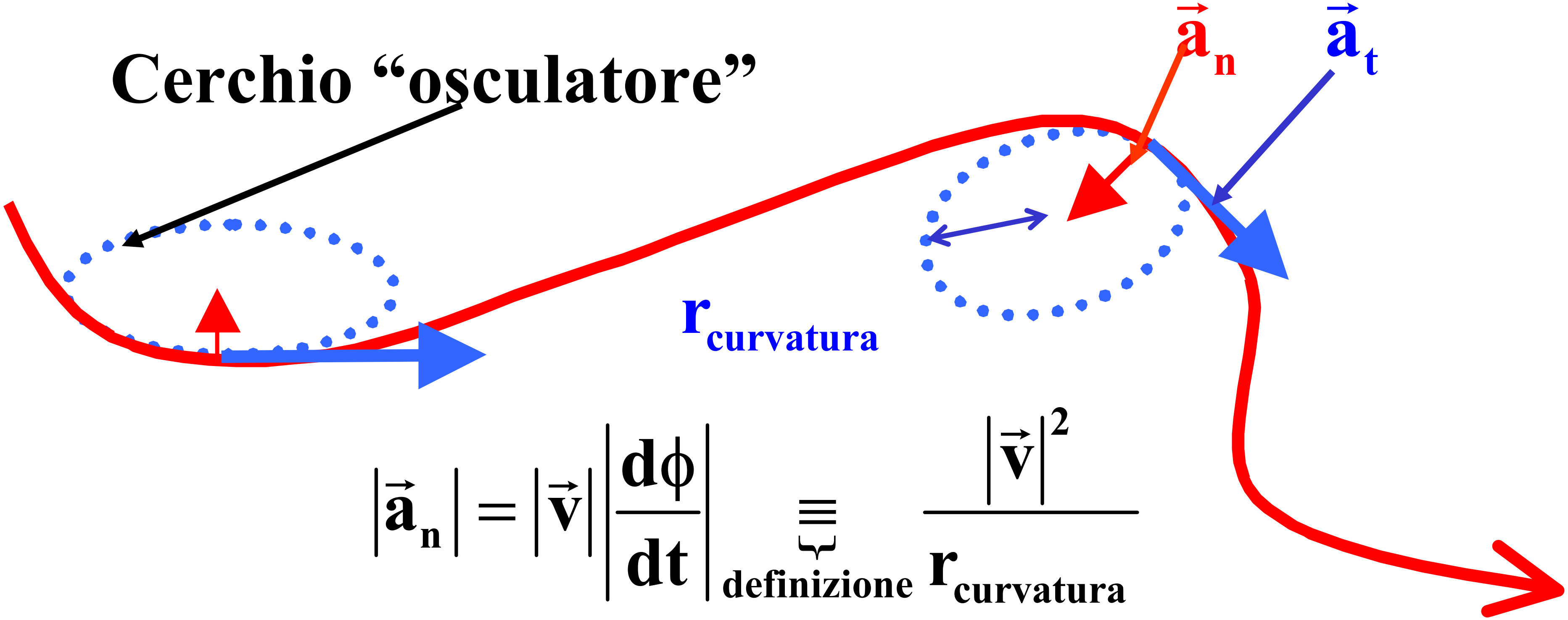
$\vec{a}_n$

$\frac{d\phi}{dt}$ → **Velocità di rotazione della velocità**

Accelerazione normale

$$|\vec{a}_n| = |\vec{v}|\left|\frac{d\phi}{dt}\right|$$

Qualunque curva localmente si può approssimare con una circonferenza



$$|\vec{a}_n| = |\vec{v}|\left|\frac{d\phi}{dt}\right| \underbrace{\equiv}_{\text{definizione}} \frac{|\vec{v}|^2}{r_{curvatura}}$$

$$\rightarrow r_{curvatura} = \frac{|\vec{a}_n|}{|\vec{v}|^2} = \frac{\left|\frac{d\phi}{dt}\right|}{|\vec{v}|}$$