Alcune Osservazioni sulla Legge di Newton
1) Dimensioni fisiche:
Massa: grandezza fondamentale
Unità SI kilogrammo (kg)

$$[\mathbf{F}] = [\mathbf{m}][\mathbf{a}] = [\mathbf{m}][\mathbf{l}][\mathbf{t}]^{-2}$$

$$\mathbf{SI} \rightarrow \mathbf{F: kg \times m \times s^{-2}} \rightarrow \mathbf{N}\ (\mathbf{Newton})$$

2) Legge di Newton e Relatività: Trasformazione di Galileo:

$$\vec{v}_P(t) = \vec{v}_P(t) + \vec{v}_o$$

$$\vec{v}_P(t) = \vec{v}_P(t) - \vec{v}_o$$

$$\vec{a}_P(t) = \frac{d\vec{v}_P(t)}{dt} = \frac{d\left(\vec{v}_P(t) + \vec{v}_o\right)}{dt} = \frac{d\vec{v}_P(t)}{dt} = \vec{a}_P(t)$$

L'accelerazione è la stessa in tutti i sistemi inerziali

Legge di Newton vale in tutti i sistemi inerziali

→ Forza uguale in tutti i sistemi inerziali

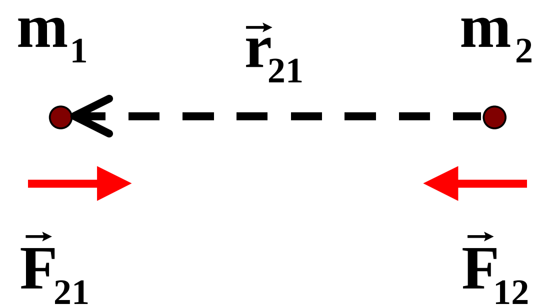
(invariante per trasformazioni di Galileo)

$$\vec{F} = m\vec{a}_P(t) = m\vec{a}_P(t) = \vec{F}$$

3) Il Catalogo: le forze fondamentali

$$\vec{F}_{21} = -G\frac{m_1 m_2}{\left|\vec{r}_{21}\right|^2}\hat{\vec{r}}_{21} = -\vec{F}_{12}$$

Forza gravitazionale



$$\vec{F} = e\left(\vec{E} + \vec{v}\times\vec{B}\right)$$

Forza elettromagnetica (di Lorentz)

$\vec{E}$ = Campo elettrico; $\vec{B}$=Campo magnetico

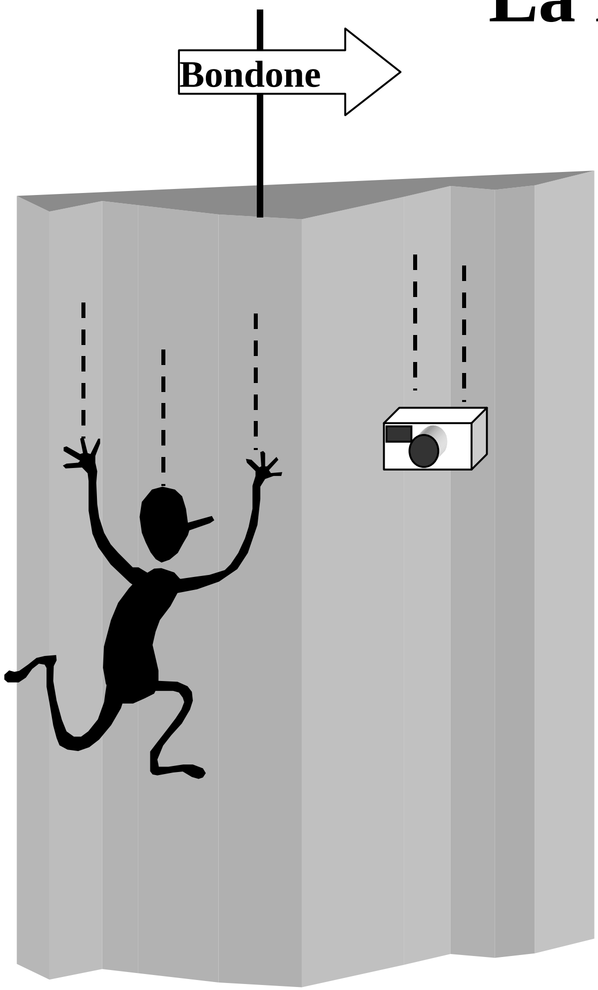
+ Interazione Nucleare Debole
= Interazione Elettro-debole

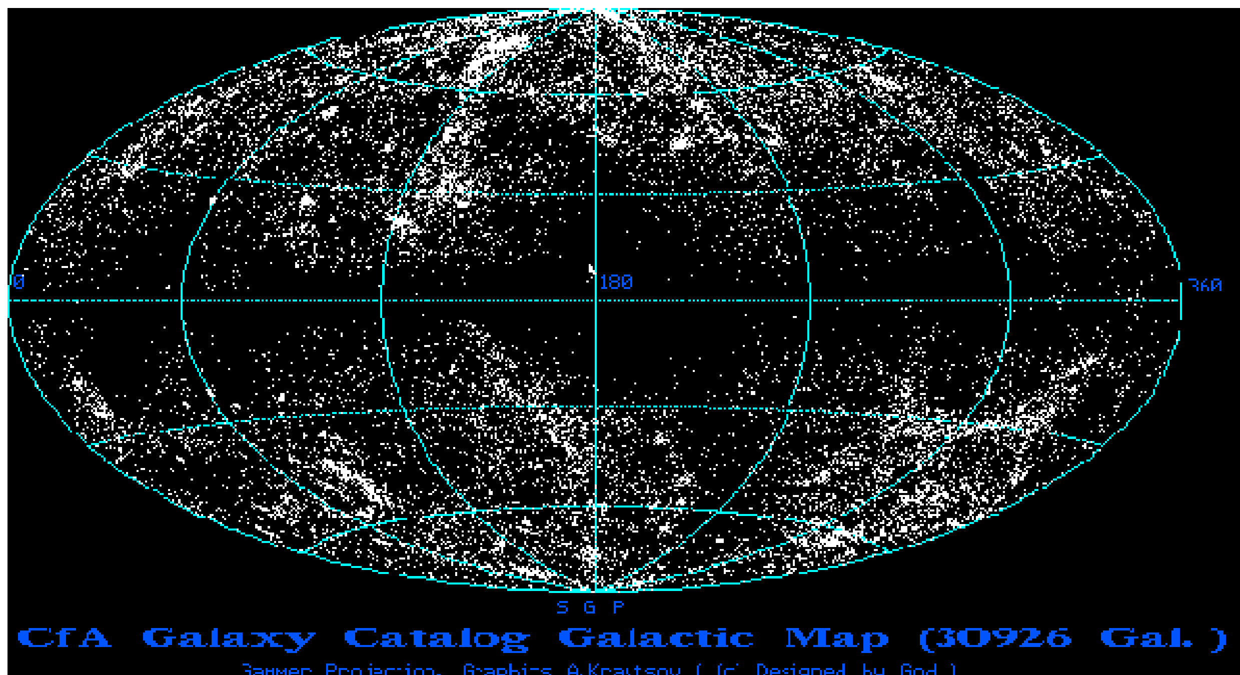
Interazione Nucleare Forte

Forza di Gravità: l'Universo

(neutralità elettrica della materia)

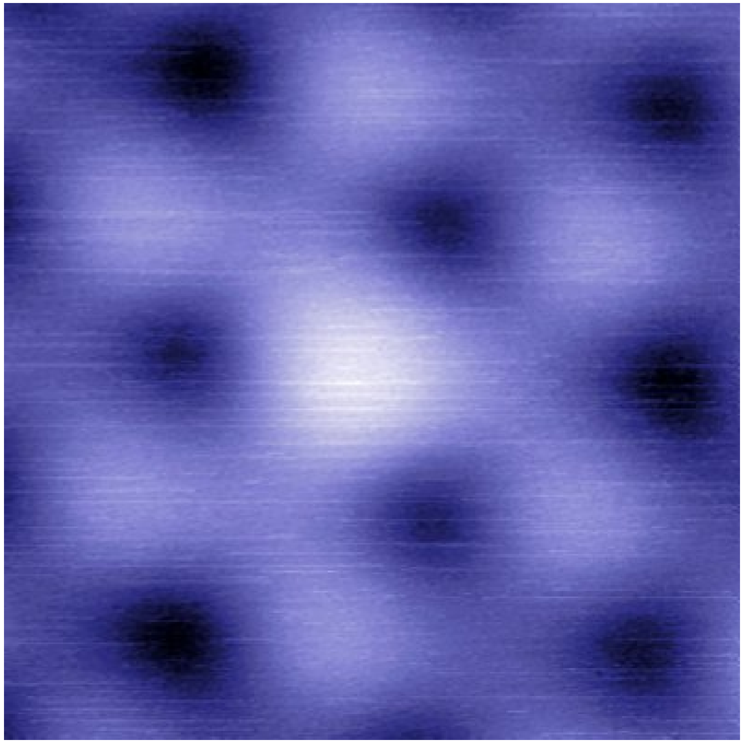
La forza peso

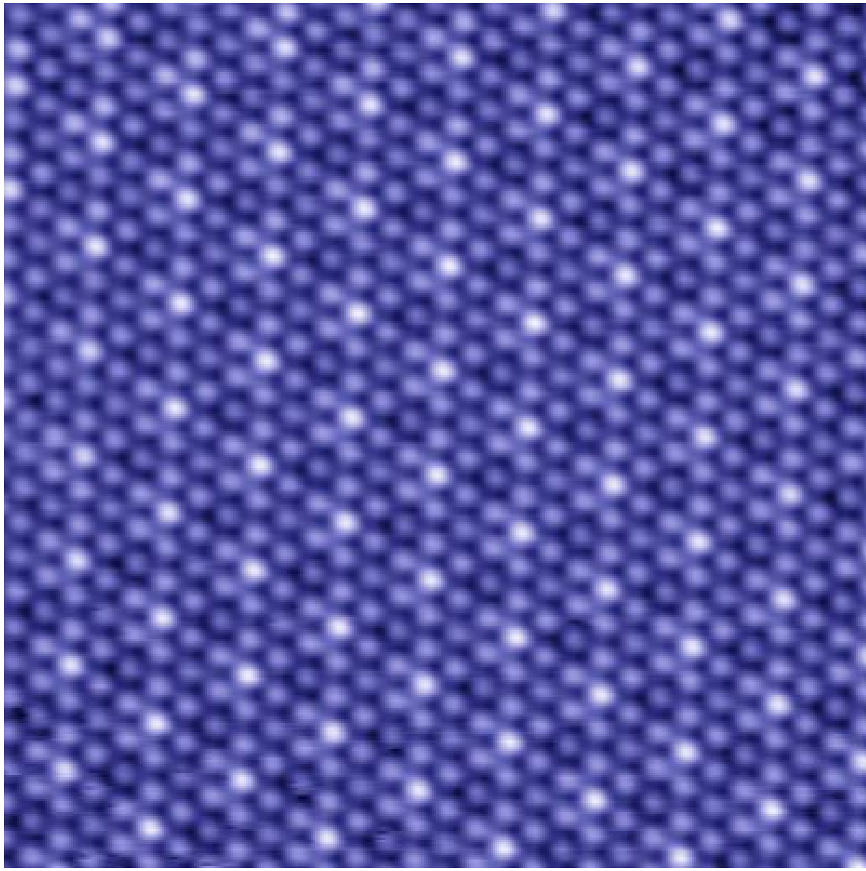




Forza Elettromagnetica

Tiene insieme elettroni e nuclei: proprietà chimiche ed elettriche della materia.



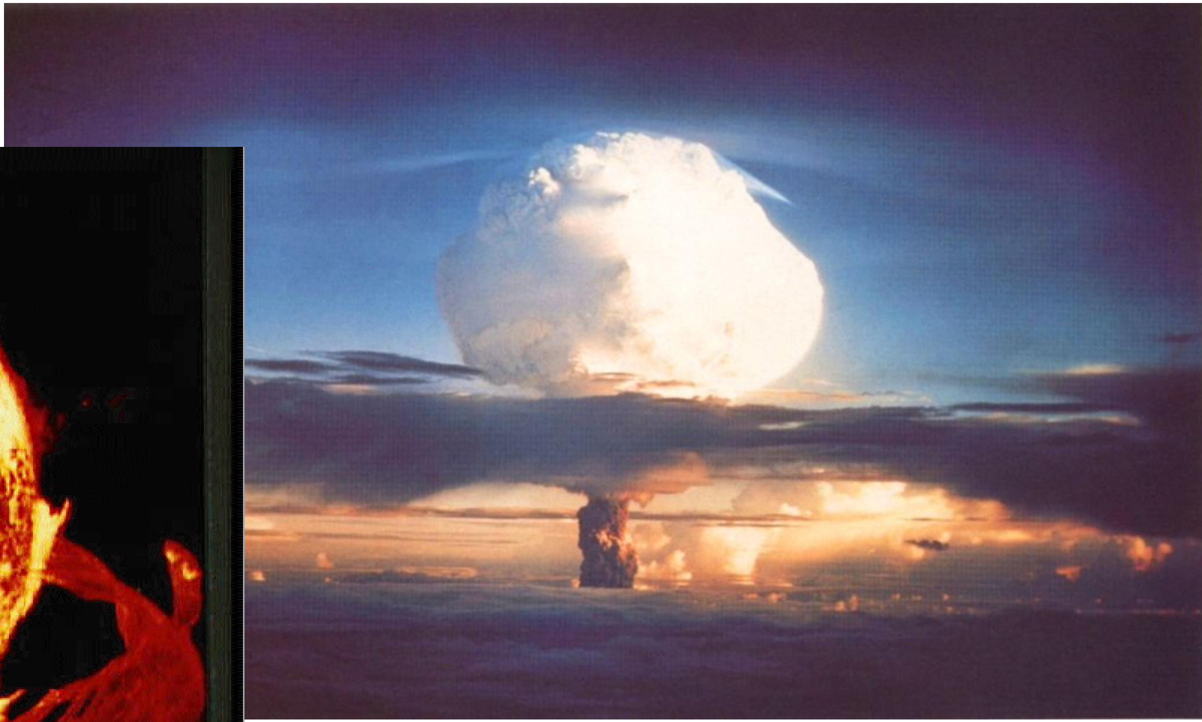


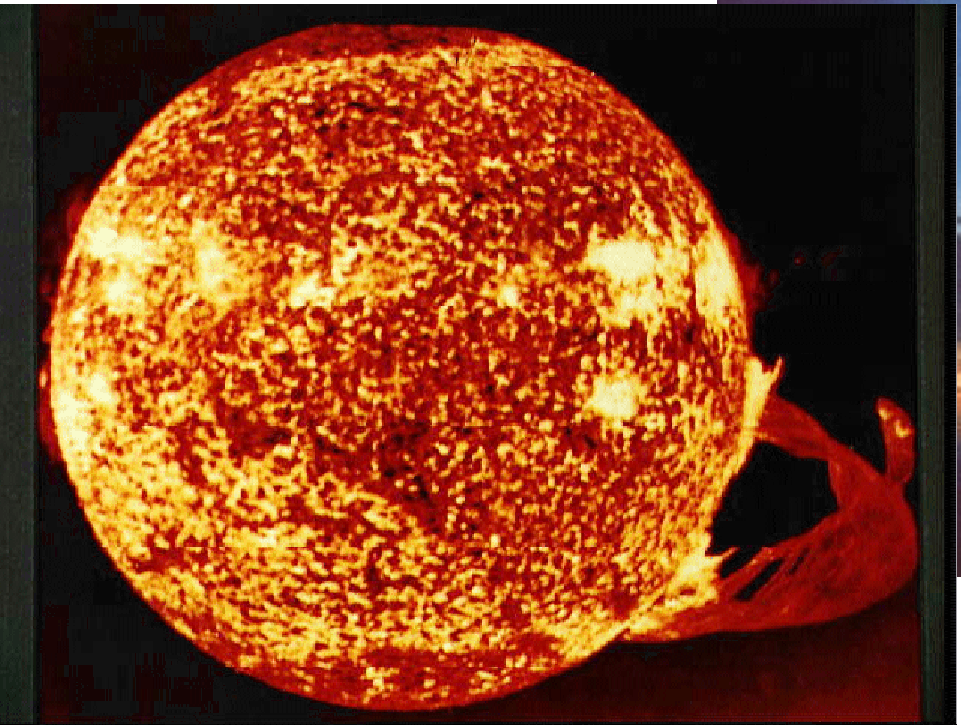
Gli atomi in un metallo ($NbSe_2$)

Interazione Nucleare Forte

Tiene insieme i nuclei

Fusione nucleare : stelle, Bomba all'idrogeno





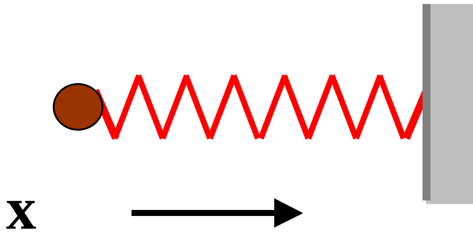
Le Forze Efficaci:

$$F_x = -k\left(x - x_o\right)$$

$$F_y,\ F_z$$

(vedi vincolo unidimensionale)

Molla in una dimensione

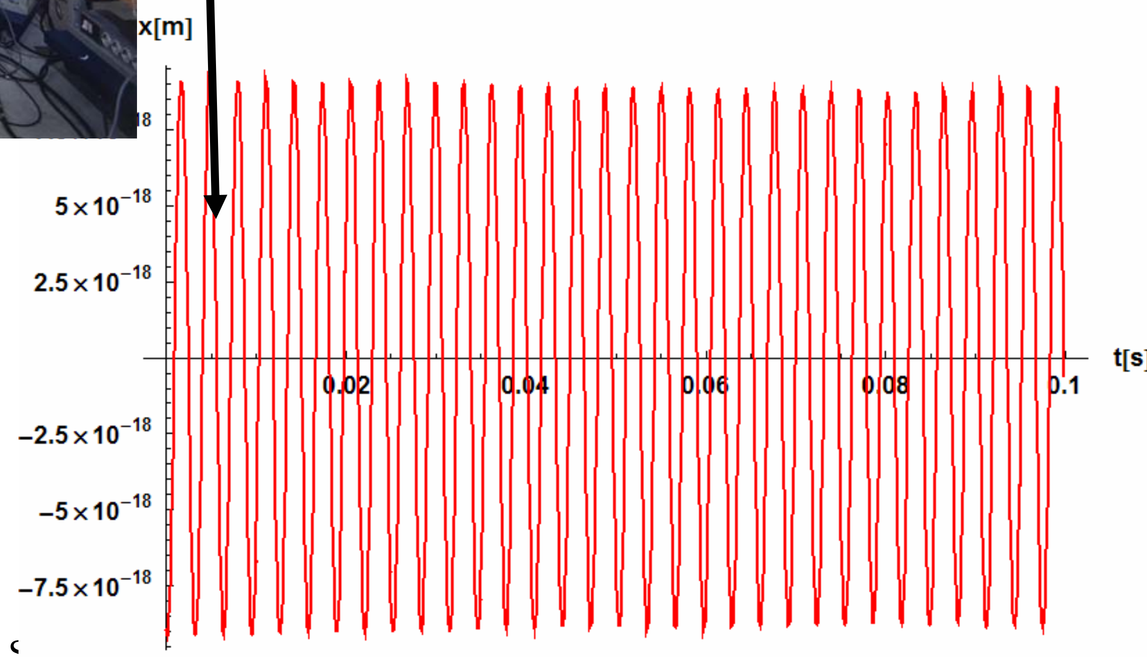


Forze elettromagnetiche fra gli atomi che compongono la molla

Cilindro in Alluminio di 2.3 Ton

a -273.05 °C

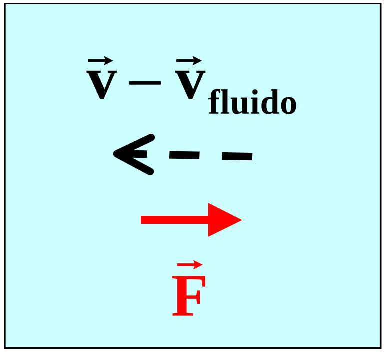
Oscillazioni della lunghezza dovute all'agitazione atomica



Attriti
(Forze elettromagnetiche fra particella e atomi di fluidi e solidi)

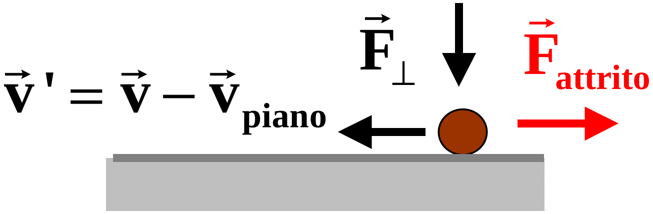
$$\vec{\mathbf{F}} = -\beta\left(\vec{\mathbf{v}} - \vec{\mathbf{v}}_{\mathbf{fluido}}\right)$$

Attrito viscoso



$$\vec{\mathbf{F}}_{\mathbf{attrito}} = -\mu_{\mathbf{d}} \left|\vec{\mathbf{F}}_{\perp}\right| \hat{\mathbf{v}}\,'$$

Attrito cinematico radente (Solo punto in movimento)

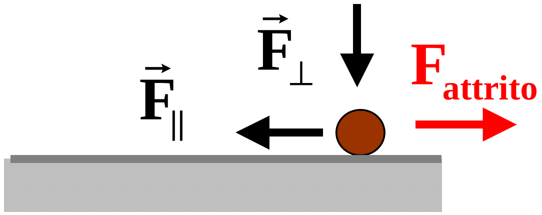


$$\vec{\mathbf{F}}_{\mathbf{attrito}} = -\vec{\mathbf{F}}_{\parallel}$$

se $\left|\vec{\mathbf{F}}_{\parallel}\right| < \mu_s \left|\vec{\mathbf{F}}_{\perp}\right|$

N.B. $\mu_s \geq \mu_{\mathbf{d}}$

Attrito statico (Solo punto in quiete)

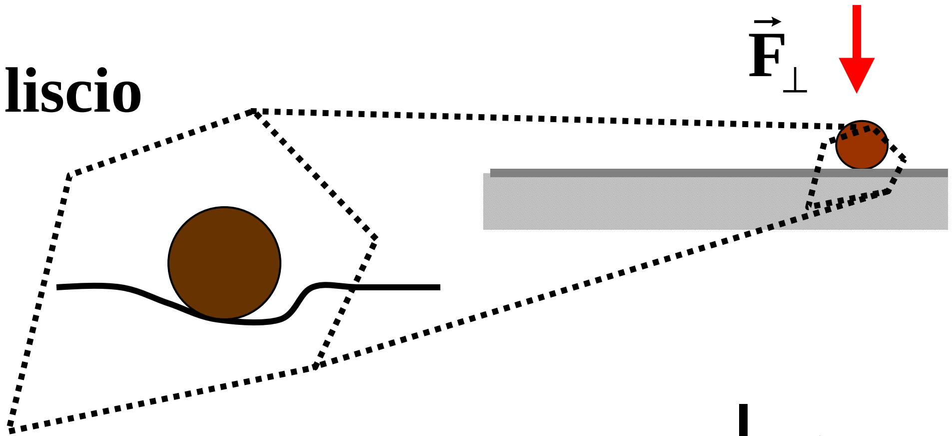


Vincoli

(Deformazioni elastiche di corpi solidi)

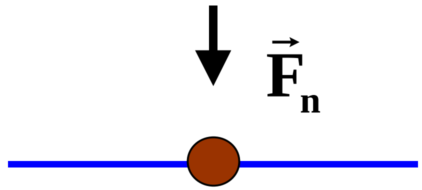
$$\vec{F}_{\perp} + \vec{F}_{piano} = 0$$

Piano liscio



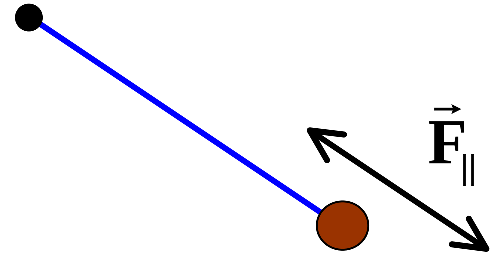
$$\vec{F}_{n} + \vec{F}_{guida} = 0$$

Guida liscia



$$\vec{F}_{\parallel} + \vec{F}_{asta} = 0$$

Asta indeformabile

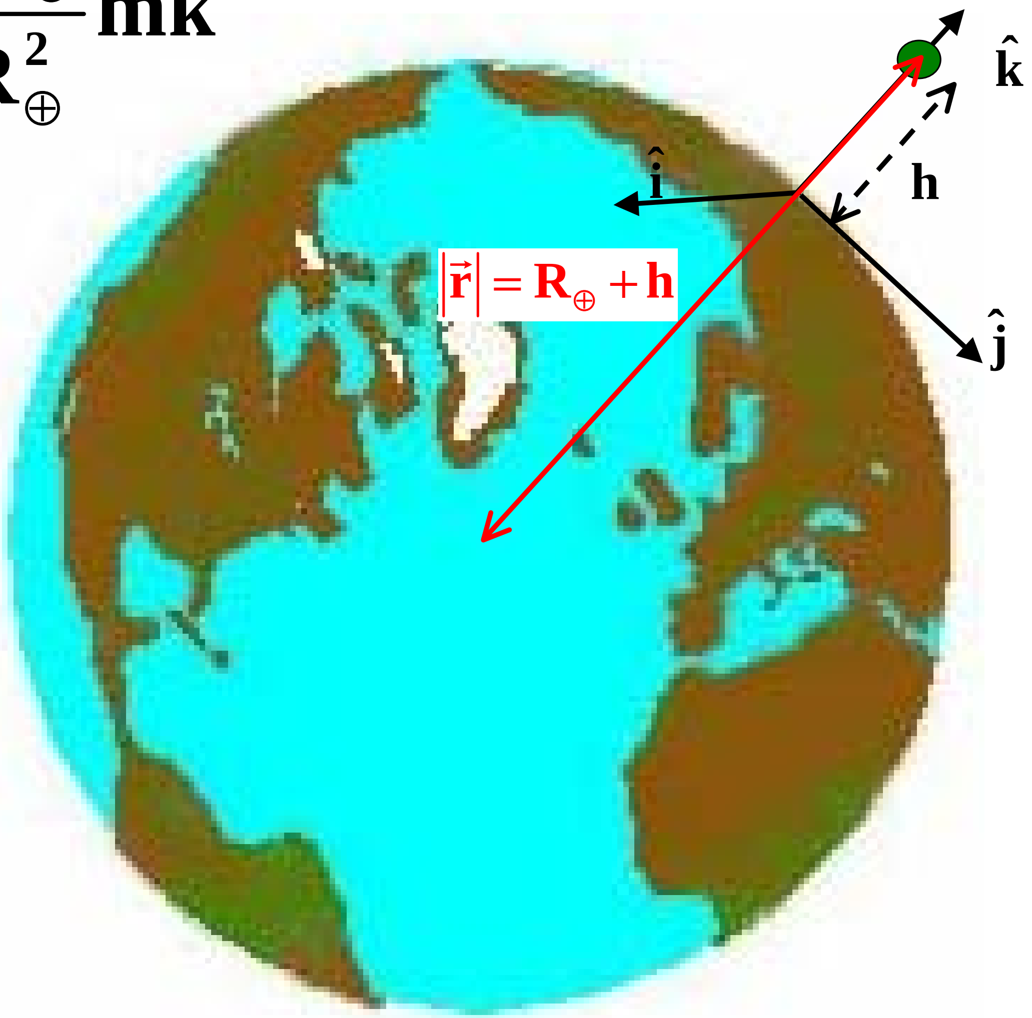


Approssimazioni di Forze Fondamentali

$$-G\frac{M_{\oplus}m}{\left(R_{\oplus}+h\right)^2}\hat{r}\approx -G\frac{M_{\oplus}}{R_{\oplus}^2}m\hat{k}$$

$$\equiv -gm\hat{k}$$

$$g=G\frac{M_{\oplus}}{R_{\oplus}^2}\approx 9.8\frac{m}{s^2}$$



N.B. Sfere equivalenti a particelle puntiformi

Forze diverse → Problemi diversi
Caso 1: forza funzione nota del tempo

$$\vec{r}(t) = \vec{r}(0) + \vec{v}(0)t + \int_0^t \left(\int_0^{t'} \frac{\vec{F}(t'')}{m} dt'' \right) dt'$$

$$x(t) = x(0) + v_x(0)t + \int_0^t \left(\int_0^{t'} \frac{F_x(t'')}{m} dt'' \right) dt'$$

$$y(t) = y(0) + v_y(0)t + \int_0^t \left(\int_0^{t'} \frac{F_y(t'')}{m} dt'' \right) dt'$$

$$z(t) = z(0) + v_z(0)t + \int_0^t \left(\int_0^{t'} \frac{F_z(t'')}{m} dt'' \right) dt'$$

Nota molto bene: I moti lungo direzioni ortogonali sono indipendenti: per conoscere y(t) non devo conoscere $F_x(t)$

Esempio 1.1 piano inclinato liscio:

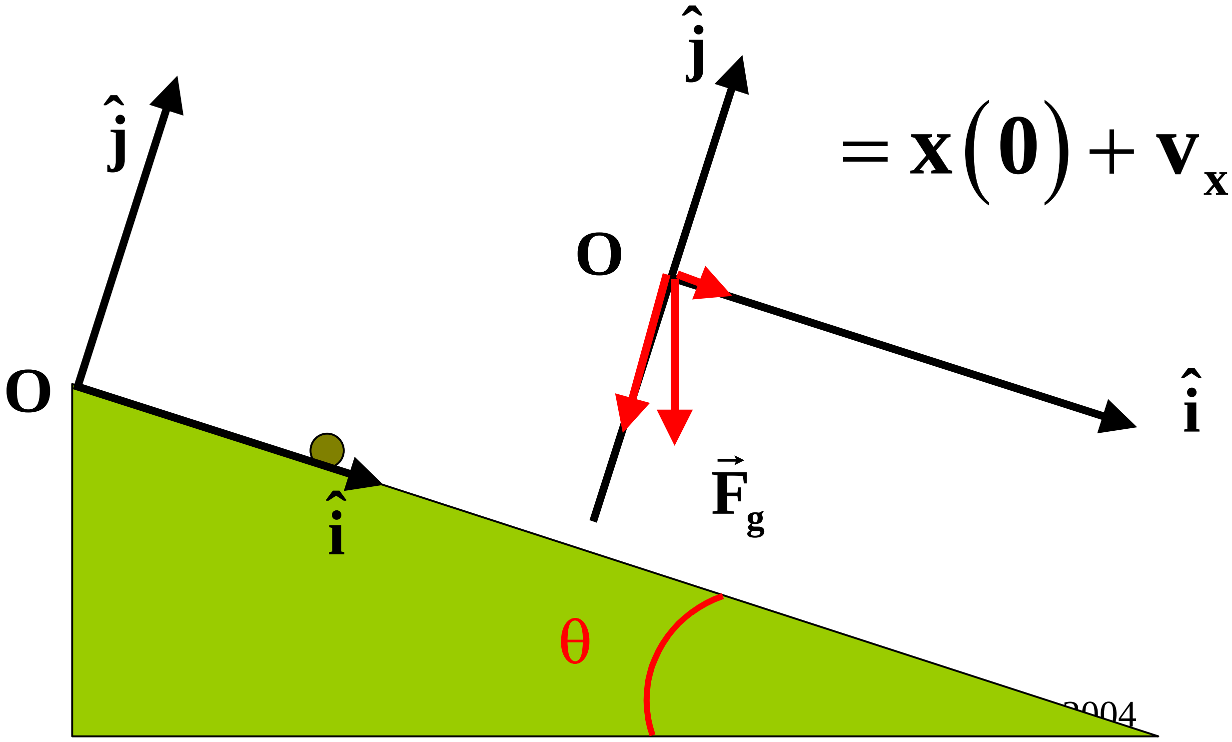
$$\vec{F}_g = mgSin(\theta)\,\hat{i} - mgCos(\theta)\,\hat{j} \qquad \hat{j} \perp \text{superfice del piano}$$

$$-mgCos(\theta) + F_{vincolo} = 0 \;\rightarrow\; \frac{dy^2}{dt^2} = 0 \text{ se } v_y(0) \text{ e } y(0) = 0 \;\rightarrow y(t) = 0$$

$$m\frac{d^2x}{dt^2} = mgSin(\theta)$$

$$\rightarrow\; x(t) = x(0) + v_x(0)t + \int_0^t \left(\int_0^{t'} gSin(\theta)\,dt'' \right) dt'$$

$$= x(0) + v_x(0)t + \frac{1}{2} gSin(\theta)\,t^2$$



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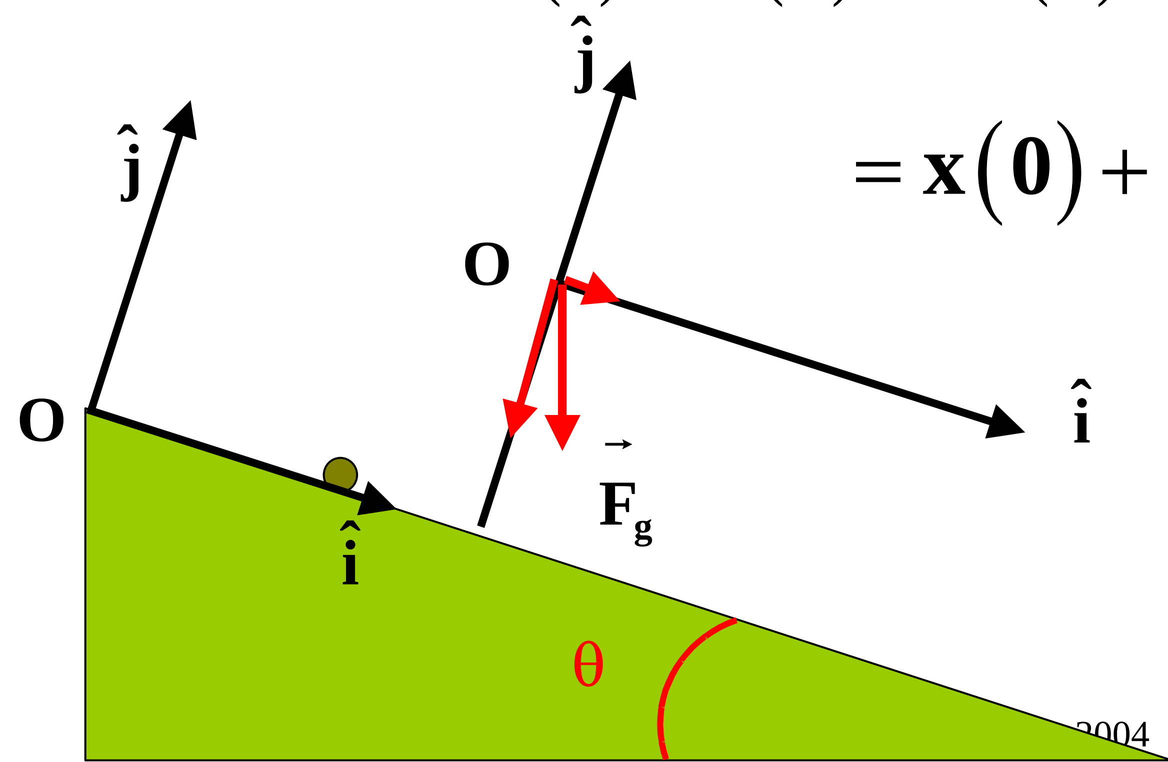
$$\rightarrow\; x(t) = x(0) + v_x(0)t + \int_0^t \left(\int_0^{t'} gSin(\theta)\,dt'' \right) dt'$$



$$= x(0) + v_x(0)t + \frac{1}{2}gSin(\theta)t^2$$

Esempio 1.2 piano inclinato con attrito:

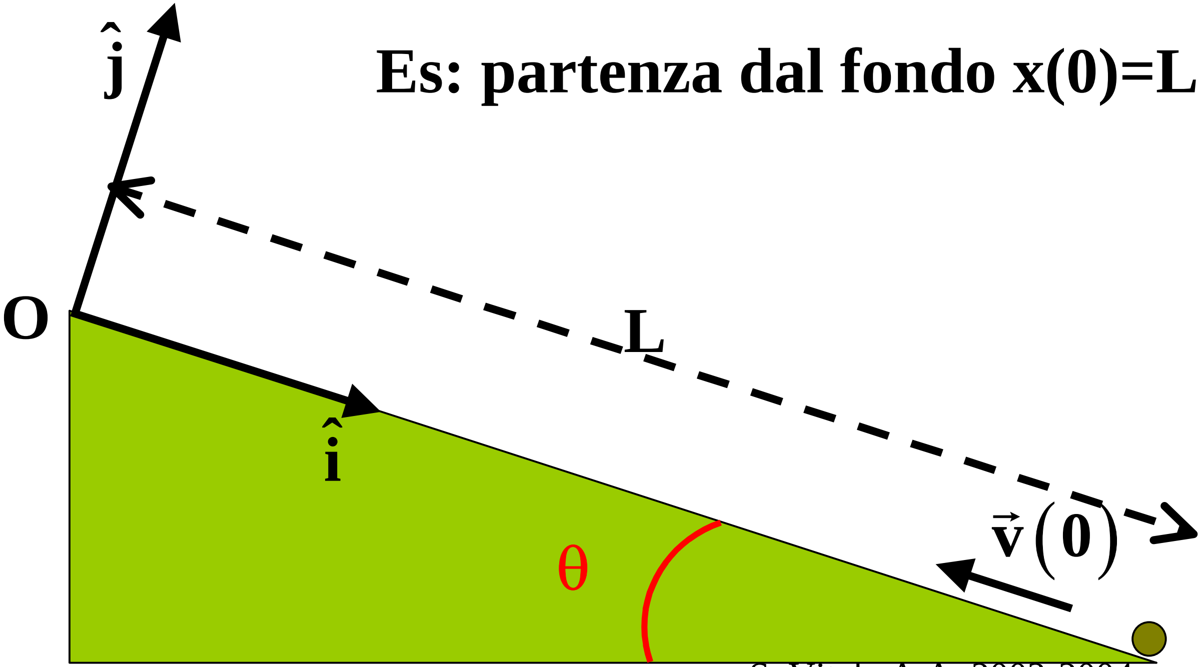
$$\left|\vec{F}_{\perp}\right| = mgCos(\theta) \quad \vec{F}_{\parallel} = mgSin(\theta)\,\hat{i}$$

$$\vec{F}_{\substack{attrito\\dinamico}} = -\mu_d mgCos(\theta)\,\hat{v} \quad \left[|\vec{v}| > 0\right]$$

$$\vec{F}_{\substack{attrito\\statico}} = -mgSin(\theta)\,\hat{i} \quad \left[|\vec{v}| = 0 \text{ e } mgSin(\theta) \le \mu_s mgCos(\theta)\right]$$

Richiede soluzione "per tentativi". Condizioni iniziali critiche

Es: partenza dal fondo x(0)=L $v_x(0)=-v_o<0$



$$m\frac{d^2x}{dt^2} = mgSin(\theta) - \mu_d mgCos(\theta)Sign[v_x(t)]$$

$$v_x(t) = -v_o + g[Sin(\theta) + \mu_d Cos(\theta)]t$$

$$\left\{t < t_{max} = \frac{v_o}{g[Sin(\theta) + \mu_d Cos(\theta)]};\ v_x(t_{max}) = 0\right\}$$

$$x(t) = L - v_o t + \frac{1}{2}g[Sin(\theta) + \mu_d Cos(\theta)]t^2$$

Si ferma o riscende?

$$x(t_{max}) = L - \frac{1}{2}\frac{v_o^2}{g[Sin(\theta) + \mu_d Cos(\theta)]}$$

$$\vec{F}_{\substack{attrito\\statico}} = -mgSin(\theta)\hat{i} \quad [|\vec{v}| = 0 \text{ e } mgSin(\theta) \le \mu_s mgCos(\theta)]$$

resta fermo se $\tan(\theta) < \mu_s$

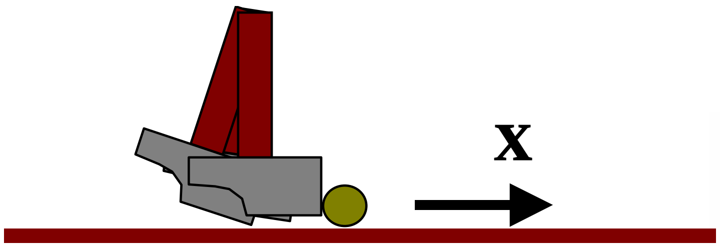
N.B. moto: $\tan(\theta) > \mu_s > \mu_d \rightarrow Sin(\theta) > \mu_d Cos(\theta)$

Esempio 1.3: forza “impulsiva”

Teorema dell’ impulso

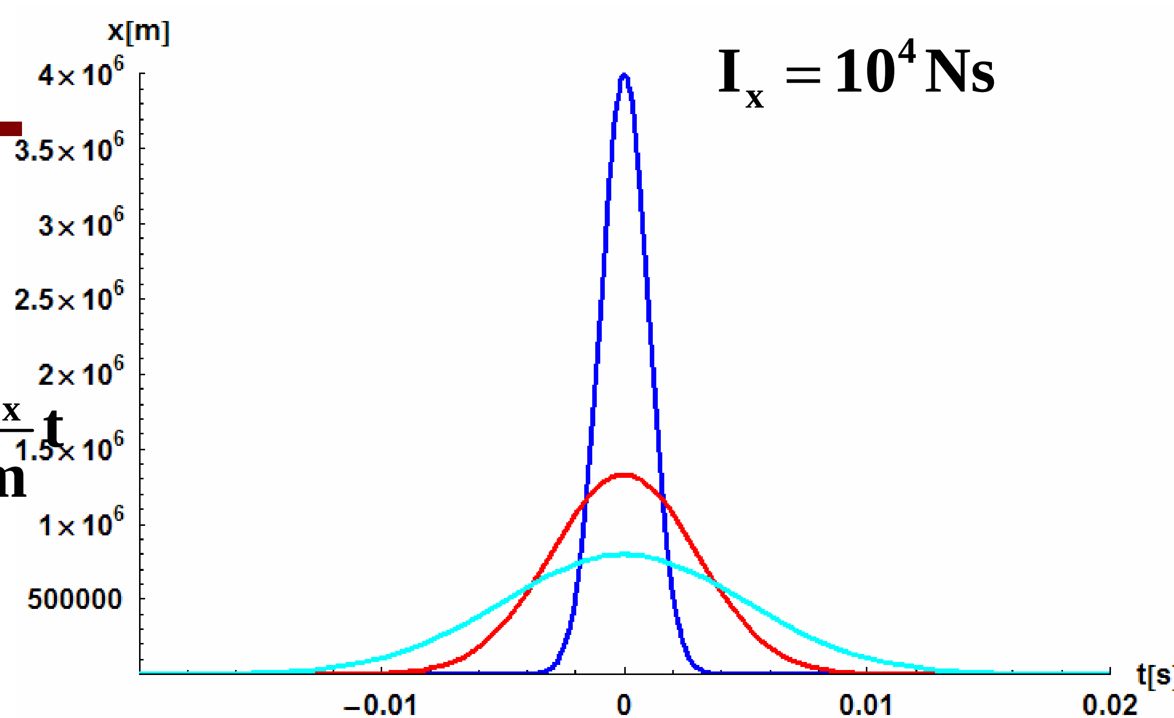
$$\vec{v}(t_2)-\vec{v}(t_1)=\int_{t_1}^{t_2}\frac{\vec{F}(t)}{m}dt \;\rightarrow\; m\vec{v}(t_2)-m\vec{v}(t_1)=\int_{t_1}^{t_2}\vec{F}(t)dt \equiv \vec{I}_{\vec{F}}(t_1,t_2)$$

$m\vec{v} \equiv$ quantità di moto $\qquad \vec{I}_{\vec{F}}(t_1,t_2) \equiv$ Impulso della forza



$$x(0),\; v_x(0)=0$$

$$v_x(t^+)=\frac{I_x}{m} \;\rightarrow\; x(t)=\frac{I_x}{m}t$$



Caso 2: forza funzione di coordinate e velocità:

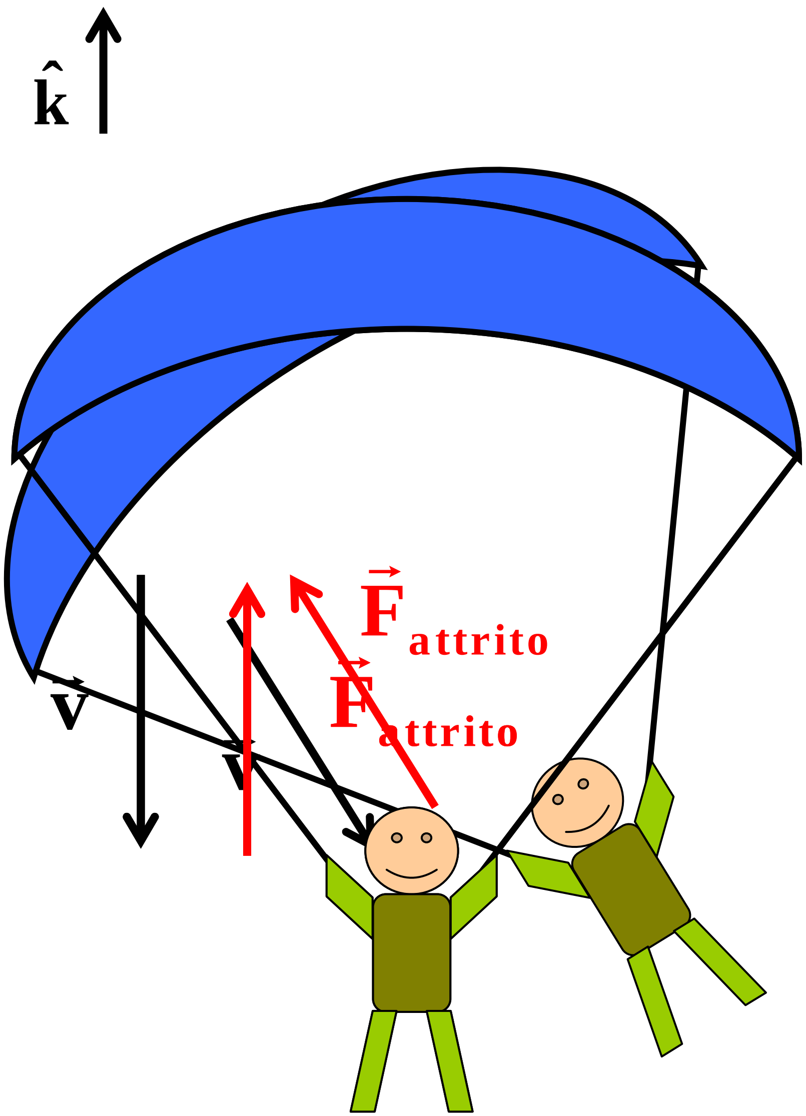
$$m\frac{d^2\vec{r}}{dt^2} = f\left[\vec{r}\left(t\right), \frac{d\vec{r}}{dt}...\right]$$

Equazione differenziale, caso “semplice”

$$m\frac{d^2\vec{r}}{dt^2} = a_o\vec{r}\left(t\right) + a_1\frac{d\vec{r}}{dt} + ..$$

Equazione differenziale, lineare a coefficienti costanti

Esempio: caduta in un fluido viscoso



$$\vec{F} = -\beta\vec{v}$$

(Fluido in quiete)

$$m\frac{d^2\vec{r}}{dt^2} = -mg\hat{k} - \beta\vec{v}$$

$$\frac{d^2x}{dt^2} + \frac{\beta}{m}\frac{dx}{dt} = 0$$

$$\frac{d^2y}{dt^2} + \frac{\beta}{m}\frac{dy}{dt} = 0$$

$$\frac{d^2z}{dt^2} + \frac{\beta}{m}\frac{dz}{dt} = -g$$

Equazione lineare non omogenea

$$\frac{d^2z}{dt^2}+\frac{\beta}{m}\frac{dz}{dt}=-g \qquad \frac{d^2z}{dt^2}+\frac{1}{\tau}\frac{dz}{dt}=-g \qquad \left(\tau=\frac{m}{\beta}\right)$$

Soluzione: 1 → trova soluzione generale dell'omogenea associata

$$\frac{d^2z}{dt^2}+\frac{1}{\tau}\frac{dz}{dt}=0 \qquad z=z_k e^{\alpha_k t} \quad \rightarrow \quad \frac{dz}{dt}=z_k\alpha_k e^{\alpha_k t} \qquad \frac{d^2z}{dt^2}=z_k\alpha_k^2 e^{\alpha_k t}$$

$$z_k\alpha_k^2 e^{\alpha_k t}+\frac{1}{\tau}z_k\alpha_k e^{\alpha_k t}=0$$

$$z_k e^{\alpha_k t}\left(\alpha_k^2+\frac{1}{\tau}\alpha_k\right)=0 \quad \rightarrow \quad \alpha_k^2+\frac{1}{\tau}\alpha_k=0 \qquad \alpha_k = \begin{matrix}0\\ -\frac{1}{\tau}\end{matrix}$$

$$z_g(t)=z_1e^{0t}+z_2e^{-\frac{t}{\tau}} \qquad =z_1+z_2e^{-\frac{t}{\tau}}$$

2→ trova una soluzione particolare (condizioni iniziali qualunque)

$$\frac{d^2z}{dt^2}+\frac{1}{\tau}\frac{dz}{dt}=-g$$

$$z_p(t)=-g\tau t \quad \rightarrow \quad \frac{dz_p}{dt}=-g\tau \quad \rightarrow \quad \frac{d^2z_p}{dt^2}=0 \quad \rightarrow \quad 0+\frac{1}{\tau}(-g\tau) \quad =-g\,!$$

3→ la soluzione generale è:

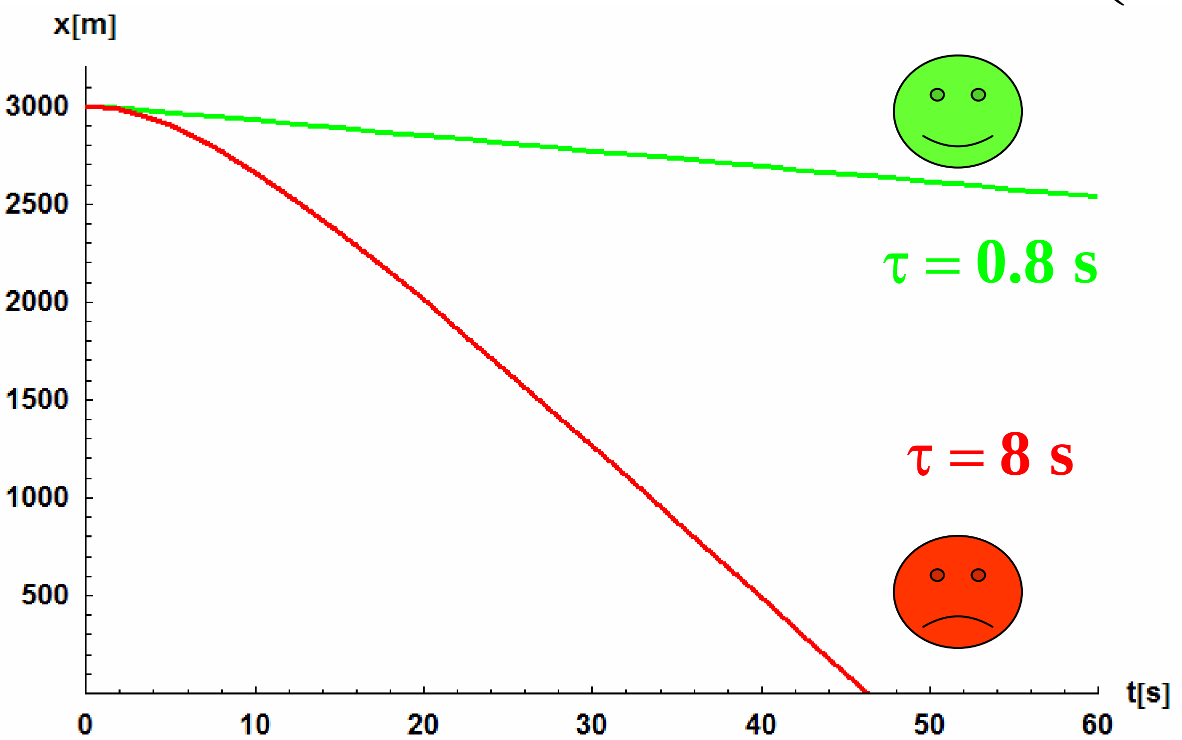
$$z(t)=z_g(t)+z_p(t)=-g\tau t \;+z_1+z_2e^{-\frac{t}{\tau}}$$

4→ Determinare z_1 e z_2 dalle condizioni inziali

$$z(0)=z_1+z_2 \qquad v_z(0)=\left[-g\tau-\frac{1}{\tau}z_2e^{-\frac{t}{\tau}}\right]_{t=0} \qquad =-g\tau-\frac{1}{\tau}z_2$$

$$z_2=-\left[v_z(0)\tau+g\tau^2\right] \quad z_1=z(0)+v_z(0)\tau+g\tau^2$$

$$z(t) = -g\tau t + z(0) + \left(v_z(0)\tau + g\tau^2\right)\left(1 - e^{-\frac{t}{\tau}}\right)$$



$$\lim_{t\to\infty} z(t) = -g\tau t \equiv -v_{limite}t$$

$$v_{limite} = g\tau$$

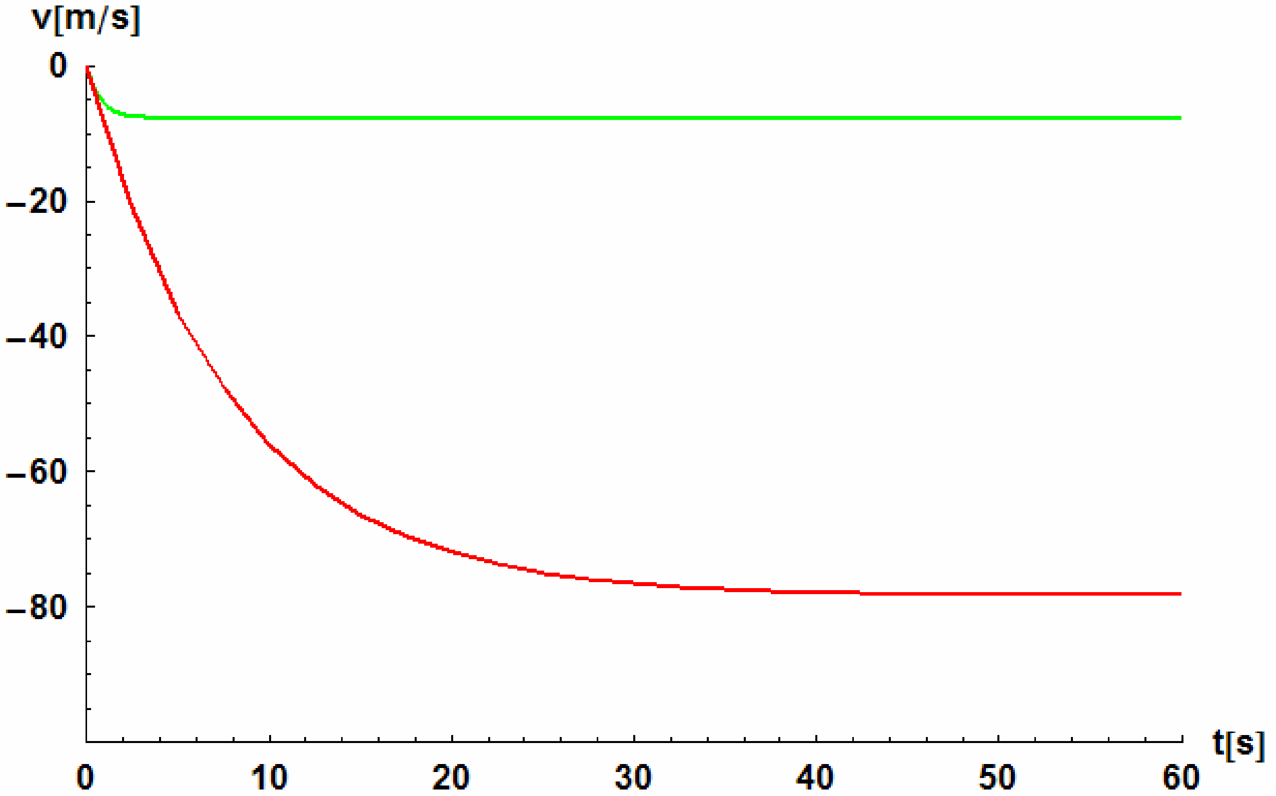
$$z(t) = -g\tau t + z(0) + \left[v(0)\tau + g\tau^2\right]\left\{1 - \left[1 - \frac{t}{\tau} + \frac{1}{2}\left(\frac{t}{\tau}\right)^2 + ..\right]\right\} =$$

$$\lim_{t\to 0} z(t) = z(0) + v(0)t - \frac{1}{2}\left[g + \frac{v(0)}{\tau}\right]t^2$$

La velocità limite

$$z(t) = -g\tau t + z(0) + \left(v_z(0)\tau + g\tau^2\right)\left(1 - e^{-\frac{t}{\tau}}\right)$$

$$v_z(t) = -g\tau + g\tau e^{-\frac{t}{\tau}} = -g\tau\left(1 - e^{-\frac{t}{\tau}}\right) = -v_{limite}\left(1 - e^{-\frac{t}{\tau}}\right)$$



$$v_{limite} = g\tau = \frac{mg}{\beta}$$

$$\beta v_{limite} = mg$$

N.B. grande τ → grande m, piccolo β (grande peso, scarso attrito)

Che succede lungo x e y?

$$\frac{d^2x}{dt^2}+\frac{1}{\tau}\frac{dx}{dt}=0$$

$$x=x_k e^{\alpha_k t} \rightarrow \frac{dx}{dt}=x_k\alpha_k e^{\alpha_k t}, \quad \frac{d^2x}{dt^2}=x_k\alpha_k^2 e^{\alpha_k t}$$

$$x_k\alpha_k^2 e^{\alpha_k t}+\frac{1}{\tau}x_k\alpha_k e^{\alpha_k t}=0 \rightarrow \alpha_k^2+\frac{1}{\tau}\alpha_k \quad \alpha_k = \begin{matrix} 0 \\ -\frac{1}{\tau} \end{matrix}$$

$$x(t)=x_1+x_2 e^{-\frac{t}{\tau}} \qquad v_x(t)=-\frac{x_2}{\tau}e^{-\frac{t}{\tau}}$$