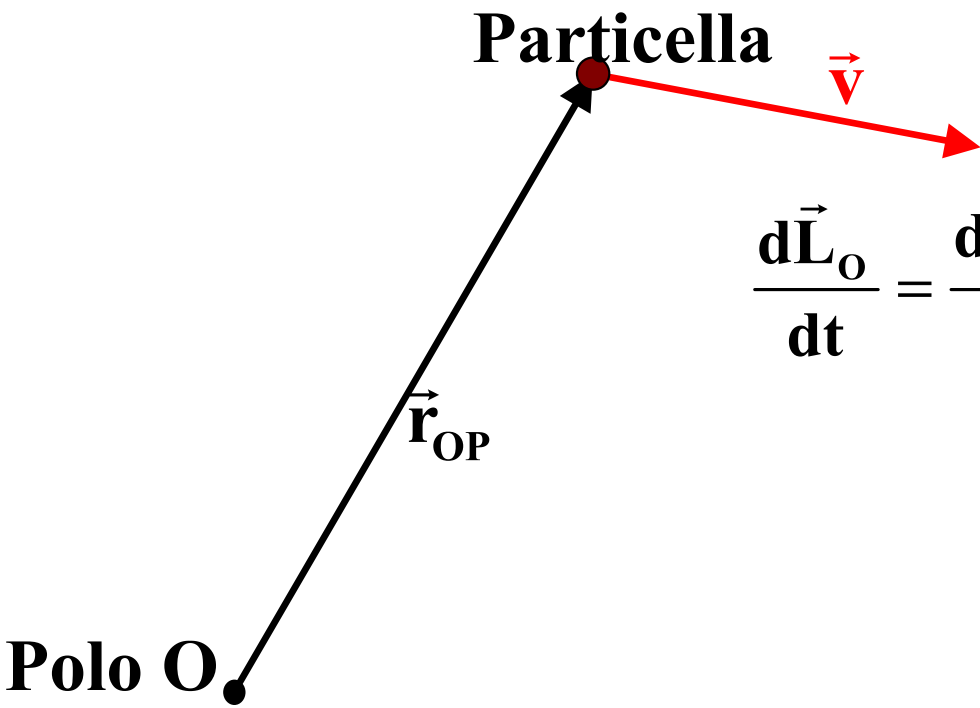
Alcune conseguenze della legge di Newton

1) Teorema del momento angolare

2) Lavoro ed energia

Teorema del momento angolare



$$\vec{L}_O = \vec{r}_{OP} \times m\vec{v} = \left(\vec{r} - \vec{r}_O\right) \times m\vec{v}$$

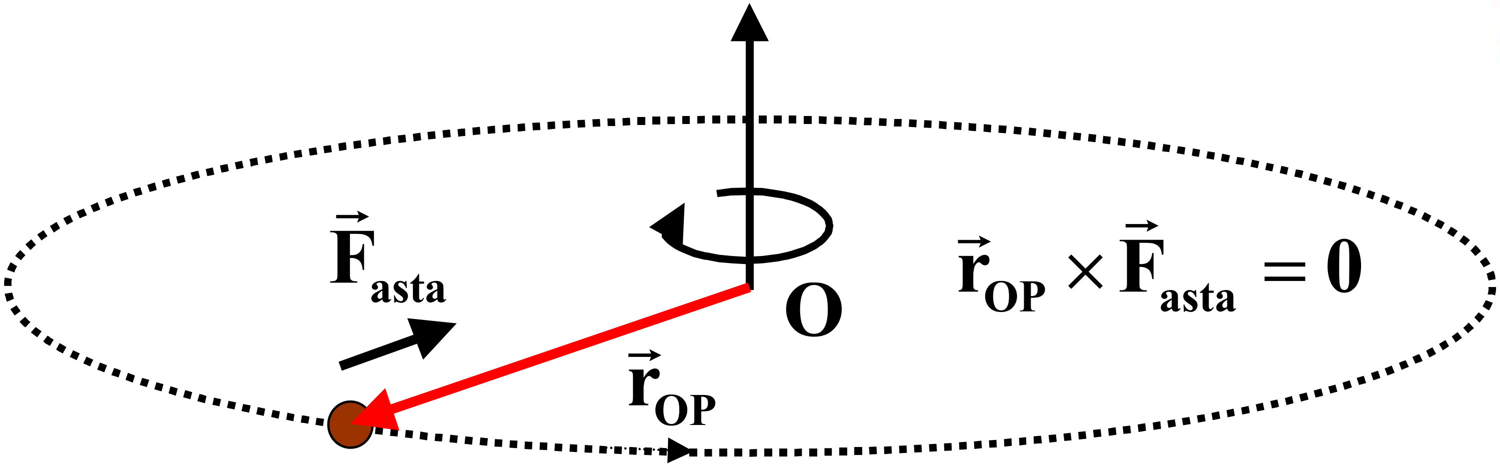
$$\frac{d\vec{L}_O}{dt} = \frac{d\left(\vec{r} - \vec{r}_O\right)}{dt} \times m\vec{v} + \left(\vec{r} - \vec{r}_O\right) \times m\frac{d\vec{v}}{dt}$$

$$== \left(\vec{v} - \vec{v}_o\right) \times m\vec{v} + \left(\vec{r} - \vec{r}_O\right) \times \vec{F}$$

$$= -\vec{v}_O \times m\vec{v} + \vec{r}_{OP} \times \vec{F}$$

Se il polo è fisso:

$$\frac{d\vec{L}_O}{dt} = \vec{r}_{OP} \times \vec{F} \equiv \vec{M}$$



$$\vec{v}(t) = -\omega r_o \mathrm{Sin}(\omega t)\,\hat{i} + \omega r_o \mathrm{Cos}(\omega t)\,\hat{j}$$

$$\vec{r}(t) = r_o \mathrm{Cos}(\omega t)\,\hat{i} + r_o \mathrm{Sin}(\omega t)\,\hat{j}$$

$$\vec{r}(t) \times m\vec{v}(t) = m\left\{\left[r_o \mathrm{Sin}(\omega t)\,\hat{j}\right] \times \left[-\omega r_o \mathrm{Sin}(\omega t)\,\hat{i}\right] + \right.$$

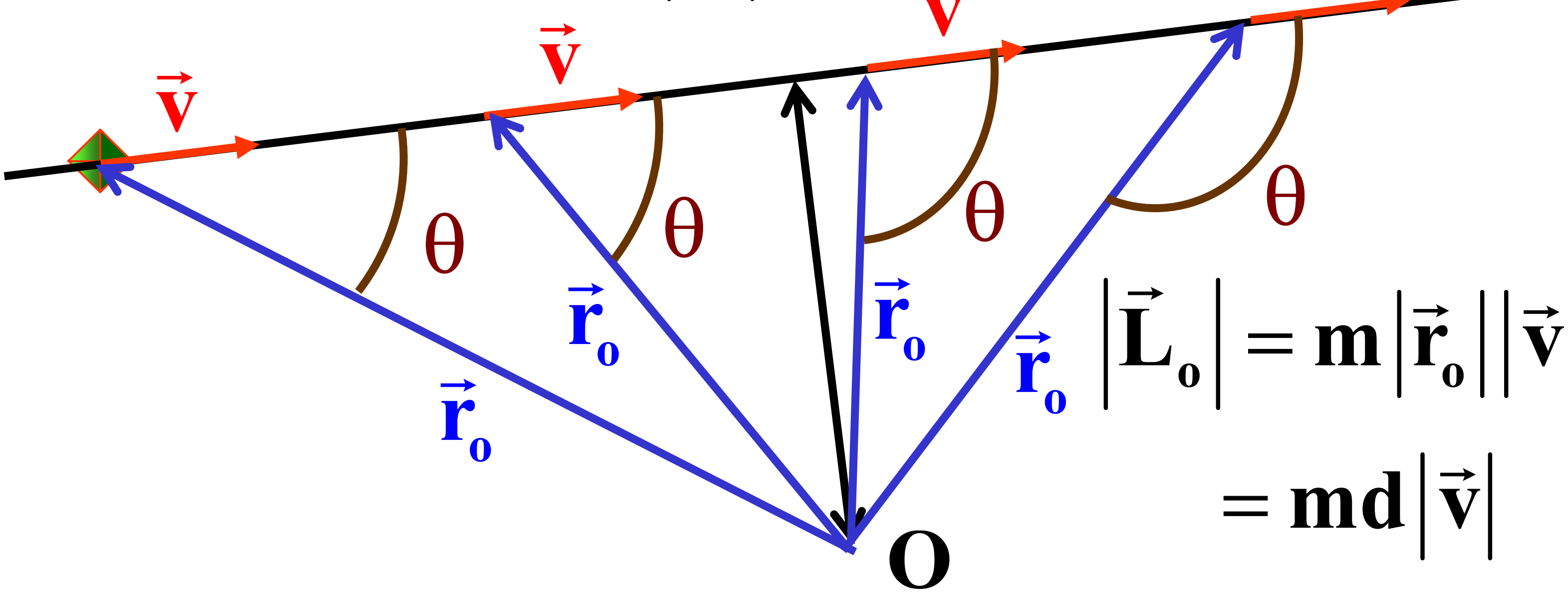
$$\left. + \left[r_o \mathrm{Cos}(\omega t)\,\hat{i}\right] \times \left[\omega r_o \mathrm{Cos}(\omega t)\,\hat{j}\right]\right\} = m\omega r_o^2 \hat{k}$$

$$\frac{d\vec{L}}{dt} = 0$$

Moto rettilineo uniforme

$$\vec{a} = 0 \qquad \vec{F} = 0 \qquad \vec{r}_o \times \vec{F} = 0 \qquad \frac{d\vec{L}_o}{dt} = 0$$

$$d = \left|\vec{r}_o\right| \mathrm{Sin}\theta$$

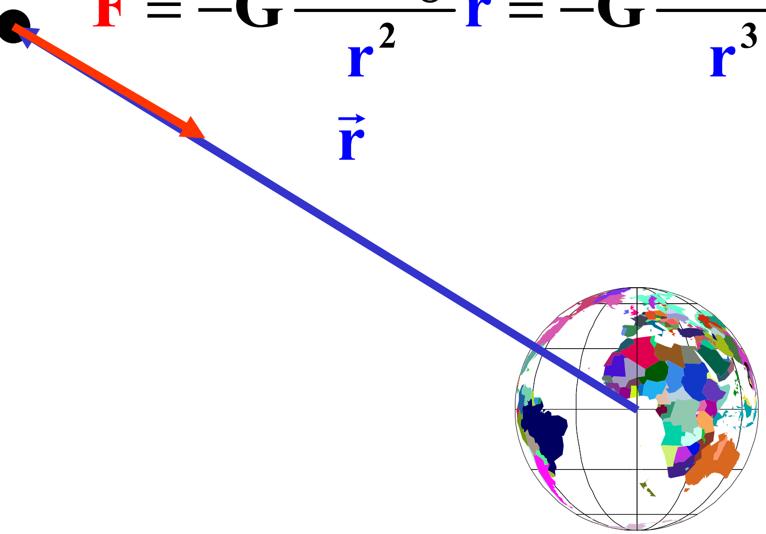


$$\left|\vec{L}_o\right| = m\left|\vec{r}_o\right|\left|\vec{v}\right| \mathrm{Sin}\theta$$

$$= md\left|\vec{v}\right|$$

La forza di gravità

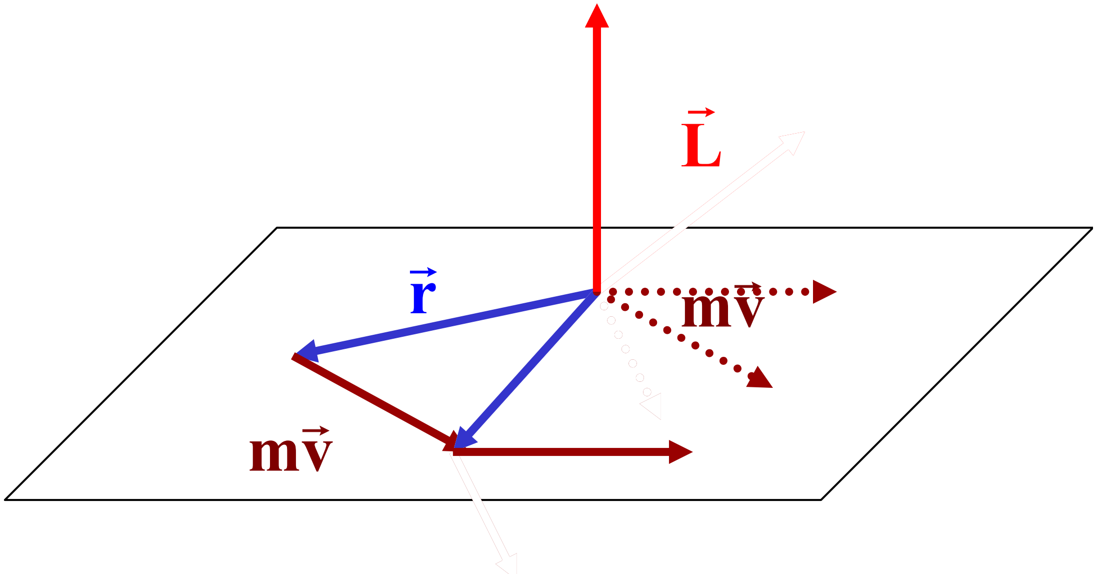
$$\vec{F} = -G\frac{mM_\oplus}{r^2}\hat{r} = -G\frac{mM_\oplus}{r^3}\vec{r}$$



$$\vec{r}\times\vec{F} = -\frac{GmM_\oplus}{r^3}\left(\vec{r}\times\vec{r}\right) = 0$$

$$G = 6.67\times 10^{-11}\frac{m^3}{kg\ s^{-2}}$$

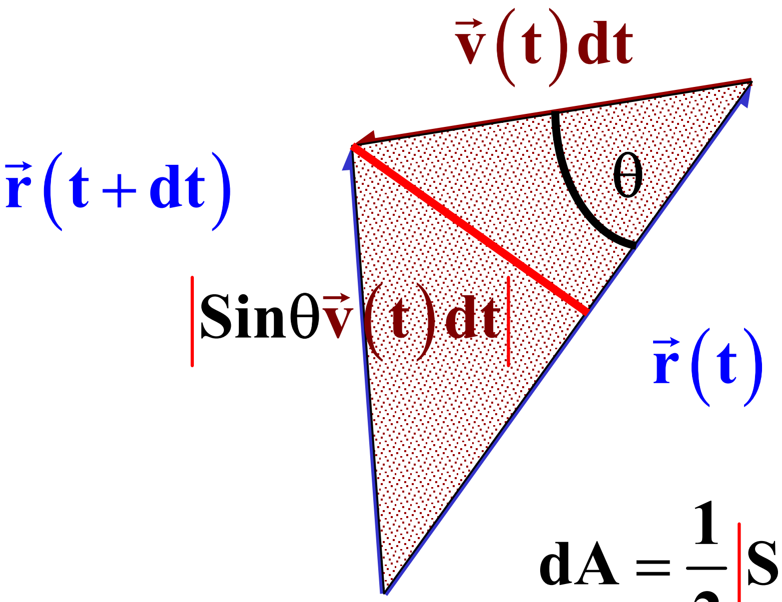
Conseguenze della conservazione del momento angolare



Si!

Il moto avviene in un piano!

Nel piano del moto:



a) l'area del triangolo disegnato dal raggio vettore che si muove

$$dA = \frac{1}{2}\left|\sin\theta\,\vec{v}(t)dt\right|\cdot\left|\vec{r}\right| \longrightarrow \frac{dA}{dt} = \frac{1}{2}\left|\sin\theta\;\vec{v}(t)\right|\cdot\left|\vec{r}\right|$$

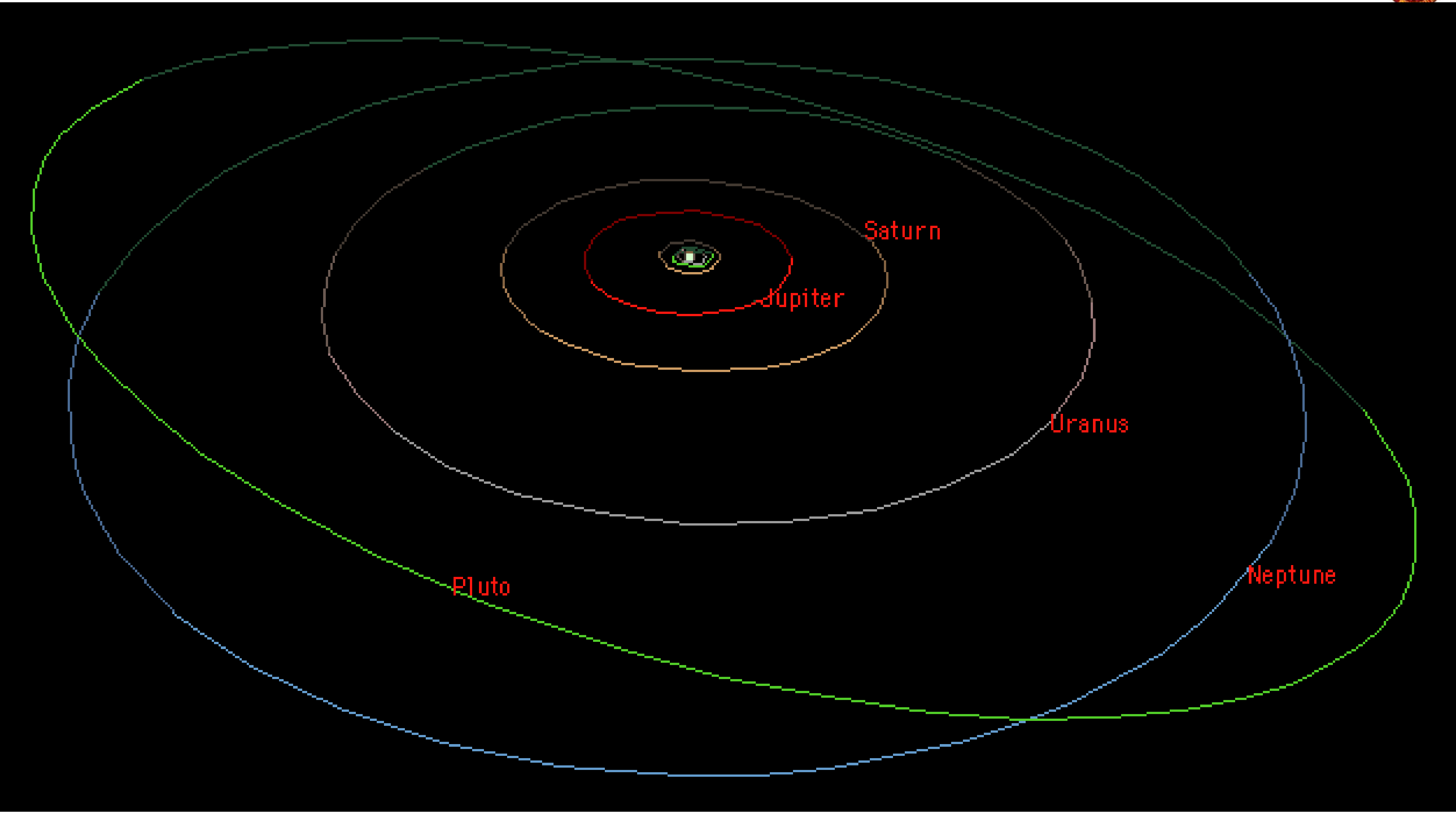
b) Il modulo del momento angolare

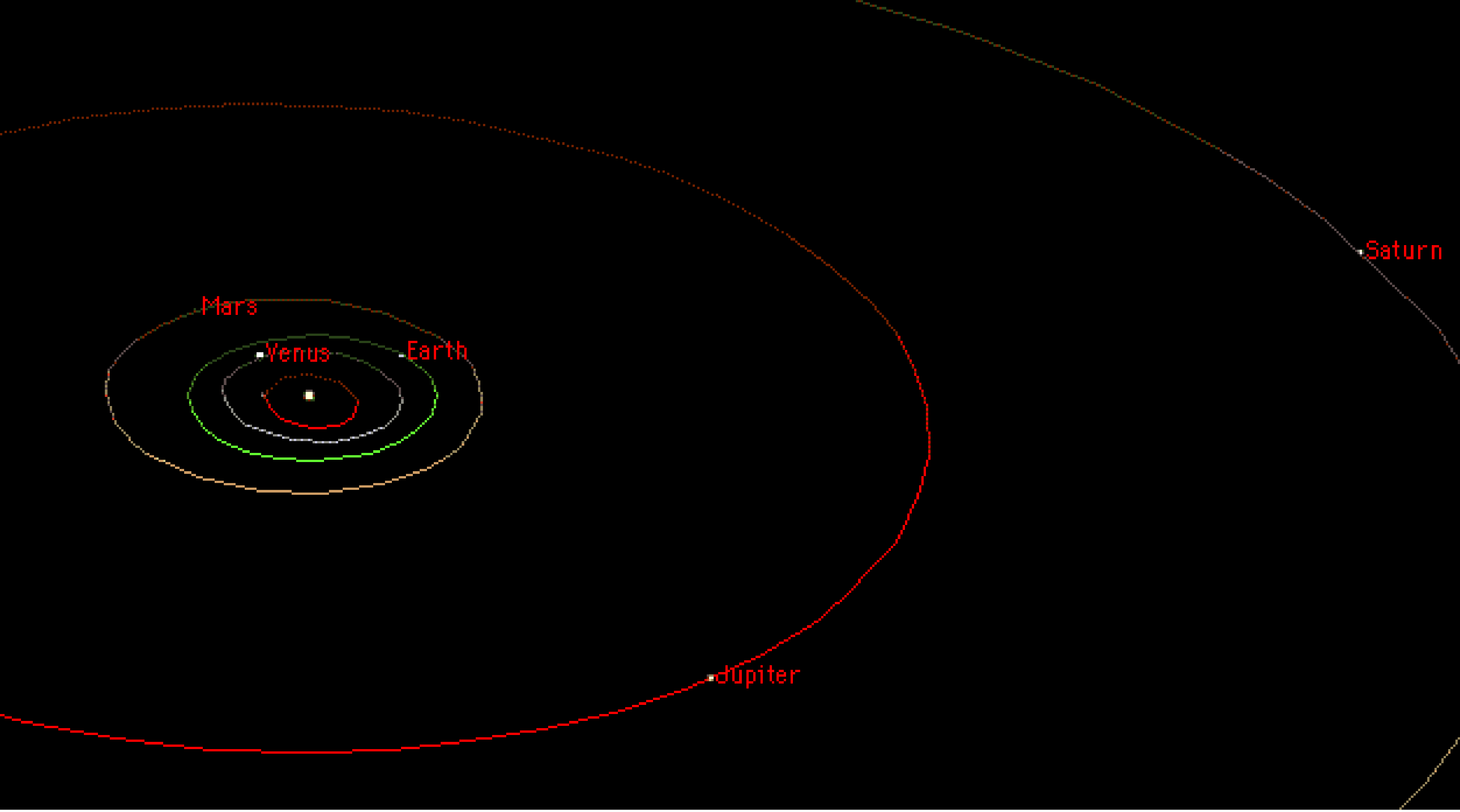
$$\left|\vec{L}\right| = m\left|\vec{r}\right|\left|\vec{v}\right|\left|\sin\theta\right|$$

a) + b) →

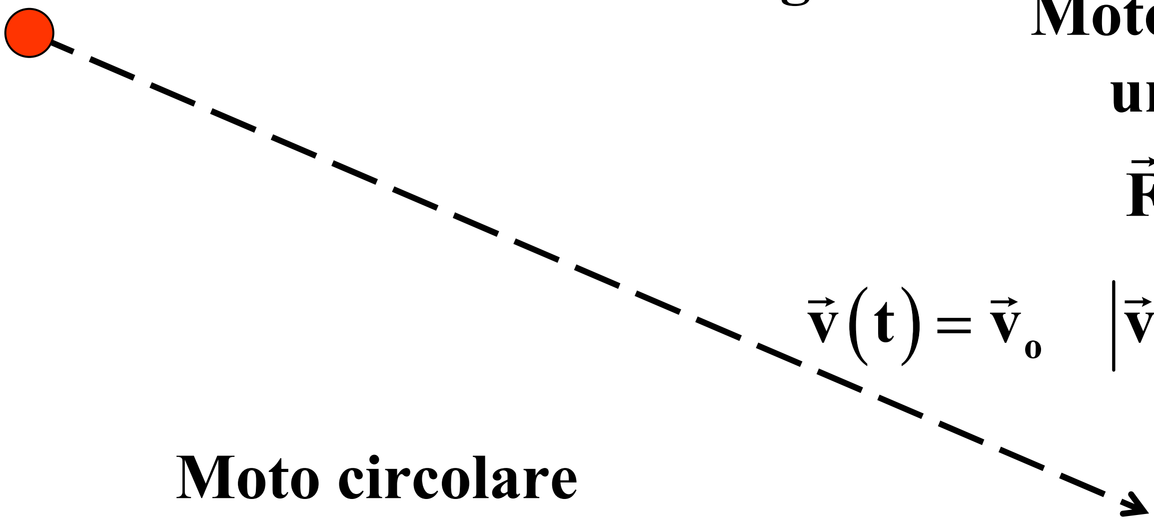
$$\frac{dA}{dt} = \frac{1}{2}\frac{\left|\vec{L}\right|}{m} = \cos t$$

La "velocità areolare è costante (Keplero)


Saturn
Jupiter
Uranus
Pluto
Neptune


Saturn
Mars
Venus
Earth
Jupiter

Lavoro ed Energia



Moto rettilineo uniforme

$$\vec{F}(t) = 0$$

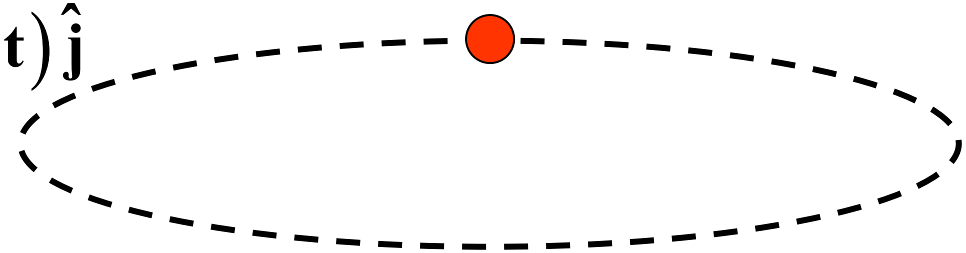
$$\vec{v}(t) = \vec{v}_o \qquad \left|\vec{v}(t)\right|^2 \equiv v^2(t) = \cos t$$

Moto circolare uniforme

$$\vec{F}(t) \neq 0$$

$$\vec{v}(t) = -r_o\omega \text{Sin}(\omega t)\,\hat{i} + r_o\omega \text{Cos}(\omega t)\,\hat{j}$$

$$\left|\vec{v}(t)\right|^2 \equiv v^2(t) = r_o^2\omega^2 = \cos t$$



Quand'è che v^2 cambia?

$$\frac{dv^2}{dt} = \frac{d\left(\vec{v}\cdot\vec{v}\right)}{dt} = \frac{d\vec{v}}{dt}\cdot\vec{v} + \vec{v}\cdot\frac{d\vec{v}}{dt} = 2\vec{v}\cdot\frac{d\vec{v}}{dt}$$

$$2\vec{v}\cdot\frac{d\vec{v}}{dt} = 2\vec{v}\cdot\vec{a} = 2\vec{v}\cdot\frac{\vec{F}}{m}$$

$$\frac{d\left(\frac{1}{2}mv^2\right)}{dt} = \vec{F}\cdot\vec{v}$$

$$\frac{1}{2}mv^2 \equiv \text{ Energia Cinetica}$$

Se la forza è nulla o ortogonale alla velocità, l'energia cinetica si conserva

Energia Cinetica: Dimensioni Fisiche

$$[m][v]^2 = [m][l]^2[t]^{-2}\left(=[F][l]\right)$$

Unità di Misura:

$$1\ \text{kg m}^2\text{s}^{-2} = 1\ \text{Joule} = 1\ \text{J}$$

$$\text{Treno in corsa} \approx \frac{1}{2}400\times10^3\ \text{kg}\times\left(50\frac{\text{m}}{\text{s}}\right)^2 = 5\times10^8\,\text{J}$$

Molecola di gas a temperatura ambiente ≈

$$\approx\frac{3}{2}k_B T = 1.5\times1.4\cdot10^{-23}\,\frac{\text{J}}{\text{K}}300\text{K}\approx 6.3\times10^{-21}\,\text{J}$$

Moto rettilineo uniforme

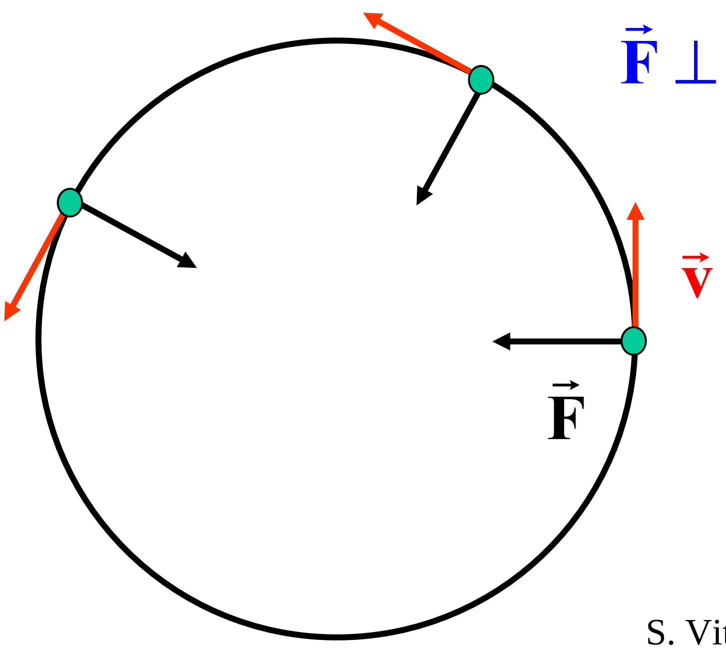
$$\vec{F}=0 \;\rightarrow\; \vec{F}\cdot\vec{v}=0 \;\rightarrow\; \frac{1}{2}mv^2=\text{cost.}$$

Moto circolare uniforme

$$\vec{F}(t)=-mr_o\omega^2\left[\text{Cos}(\omega t)\hat{i}+\text{Sin}(\omega t)\hat{j}\right]$$

$$\vec{v}(t)=r_o\omega\left[-\text{Sin}(\omega t)\hat{i}+\text{Cos}(\omega t)\hat{j}\right]$$

$$\vec{F}\perp\vec{v} \;\rightarrow\; \vec{F}\cdot\vec{v}=0 \;\rightarrow\; \frac{1}{2}mv^2=\frac{1}{2}mr_o^2\omega^2$$



Teorema dell'energia cinetica

$$\frac{d\left(\frac{1}{2}mv^2\right)}{dt} = \vec{F}\cdot\vec{v}$$

Versione integrale:

$$\frac{1}{2}mv^2(t_B) - \frac{1}{2}mv^2(t_A) = \int_{t_A}^{t_B} \vec{F}(t')\cdot\vec{v}(t')\,dt' =$$

$$= \int_{t_A}^{t_B} \left[F_x(t')v_x(t') + F_y(t')v_y(t') + F_z(t')v_z(t')\right]dt'$$

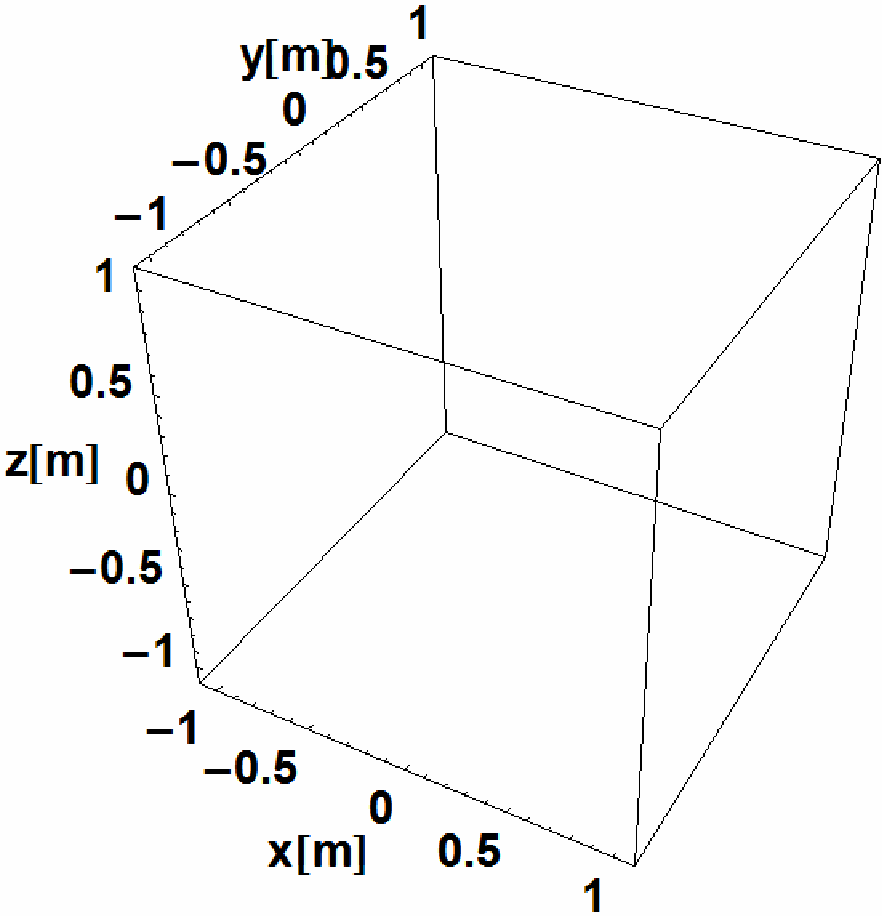
Un intervallo di tempo infinitesimo

$$\frac{1}{2}mv^2(t+\delta t)-\frac{1}{2}mv^2(t)=\int_t^{t+\delta t}\vec{F}(t')\cdot\vec{v}(t')dt' \approx \vec{F}(t)\vec{v}(t)\delta t$$

ma $\quad \vec{v}(t)\delta t \approx \frac{\vec{r}(t+\delta t)-r(t)}{\delta t}\delta t = \vec{r}(t+\delta t)-r(t) = d\vec{r}(t,t+\delta t)$

$$\frac{1}{2}mv^2(t+\delta t)-\frac{1}{2}mv^2(t)=\vec{F}(t)\cdot d\vec{r}(t,t+\delta t)$$

Lavoro elementare ⟵ $dL \equiv \vec{F}\cdot d\vec{r} = d\left(\frac{1}{2}mv^2\right)$



$$x(t) = (1\ \mathrm{m}) \cdot \mathrm{Cos}\left(1 \frac{\mathrm{rad}}{\mathrm{s}}\ t\right)$$

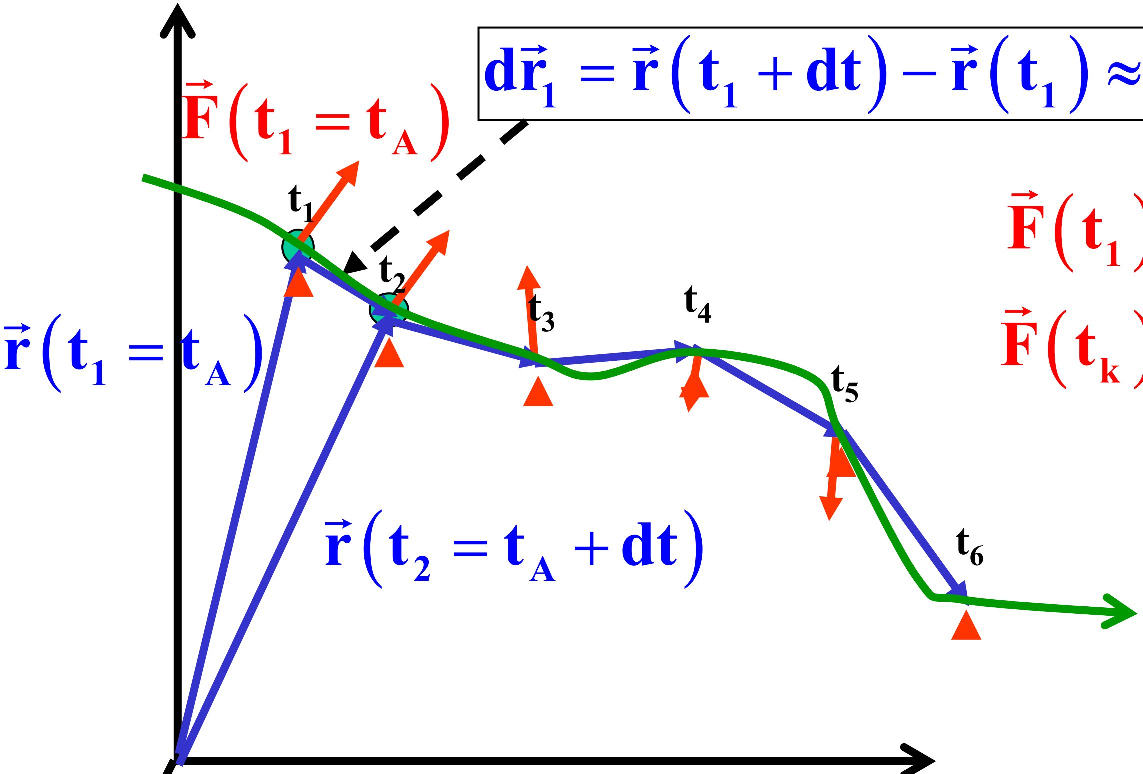
$$y(t) = (1\ \mathrm{m}) \cdot \mathrm{Sin}\left(1 \frac{\mathrm{rad}}{\mathrm{s}}\ t\right)$$

$$z(t) = 0.1 \frac{\mathrm{m}}{\mathrm{s}}\ t$$

Equazione parametrica di una curva:

Mentre il parametro “t” scorre x,y e z disegnano una curva: la traiettoria

La variazione di energia come “integrale di linea” della forza



$$\vec{F}(t_1)\cdot\vec{v}(t_1)\,dt = \vec{F}_1\cdot d\vec{r}_1$$

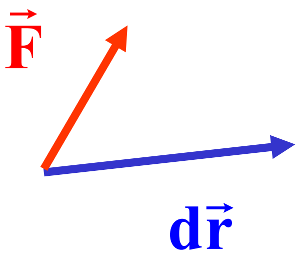
$$\vec{F}(t_k)\cdot\vec{v}(t_k)\,dt = \vec{F}_k\cdot d\vec{r}_k$$

$$\frac{1}{2}mv^2(t_B) - \frac{1}{2}mv^2(t_A) = \int_{t_A}^{t_B}\vec{F}(t)\cdot\vec{v}(t)\,dt = \lim_{N\to\infty}\left(\sum_{k=1}^{N-1}\vec{F}(t_k)\cdot\vec{v}(t_k)\,dt\right)$$

$$= \lim_{N\to\infty}\left(\sum_{k=1}^{N-1}\vec{F}_k\cdot d\vec{r}_k\right) \equiv \int_{\text{Traiettoria}}\vec{F}\cdot d\vec{r}$$

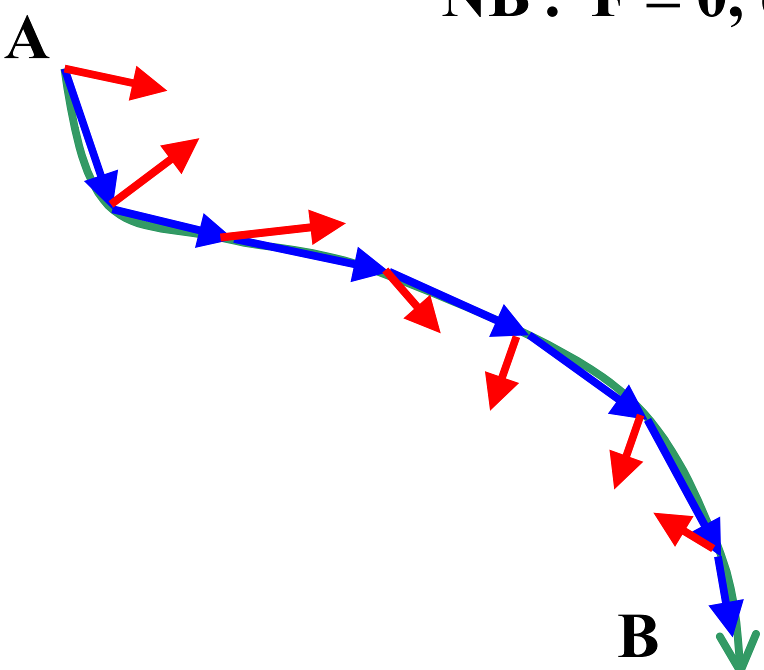
Una definizione: il lavoro fatto da una forza

1 Lavoro elementare



$$dL = \vec{F} \cdot d\vec{r} = F_x dx + F_y dy + F_z dz$$

$$NB: \ \vec{F} = 0, \ d\vec{r} = 0, \ \vec{F} \perp d\vec{r} \ \rightarrow \ dL = 0$$



2 Lavoro "finito" lungo una curva:

Somma dei lavori infinitesimi

$$L_{A\rightarrow B} = \lim_{N\rightarrow\infty} \sum_{k=1}^{N} dL_k = \lim_{N\rightarrow\infty} \sum_{k=1}^{N} \vec{F}_k d\vec{r}_k$$

Se sul punto agisce più di una forza:

$$\vec{F}_{tot} = \vec{F}_1 + \vec{F}_2 + ... + \vec{F}_n$$

$$L_{tot,A\to B} = \underset{N\to\infty}{\mathrm{Lim}} \sum_{k=1}^{N} \vec{F}_{tot,k} \cdot dr_k = \underset{N\to\infty}{\mathrm{Lim}} \sum_{k=1}^{N} \left(\vec{F}_{1,k} + \vec{F}_{2,k} + ... + \vec{F}_{n,k}\right) \cdot dr_k =$$

$$\underset{N\to\infty}{\mathrm{Lim}} \sum_{k=1}^{N} \vec{F}_{1,k} \cdot dr_k + \underset{N\to\infty}{\mathrm{Lim}} \sum_{k=1}^{N} \vec{F}_{2,k} \cdot dr_k + .. + \underset{N\to\infty}{\mathrm{Lim}} \sum_{k=1}^{N} \vec{F}_{n,k} \cdot dr_k =$$

$$= L_{1,A\to B} + L_{2,A\to B} + ... + L_{N,A\to B}$$

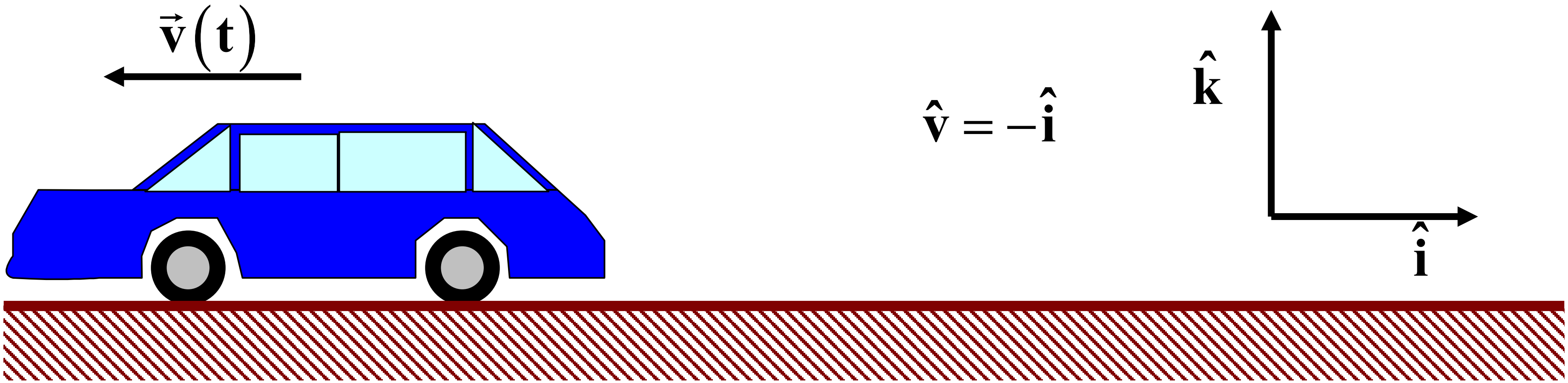
In definitiva: il teorema dell'energia cinetica

$$\frac{1}{2} m v^2 (t_B) - \frac{1}{2} m v^2 (t_A) = L_{tot,A\to B}$$

Esempio: frenata per attrito radente

(auto con ruote bloccate)

$$\vec{F} = -\mu_d mg\hat{v} + \vec{F}_{vincolo} - mg\hat{k} = -\mu_d mg\hat{v}$$



$$\hat{v} = -\hat{i}$$

$$v_x(t) = -|v_x(0)| + \mu_d gt \quad x(t) = -|v_x(0)|t + \frac{1}{2}\mu_d gt^2$$

$$t_A = 0 \;\rightarrow\; x_A = 0 \quad t_B = \frac{|v_x(0)|}{\mu_d g} \;\rightarrow\; v_x(t_B) = 0 \;\rightarrow\; x_B = -\frac{v_x^2(0)}{2\mu_d g}$$

$$L_{attrito,A\rightarrow B} = \int_A^B \mu_d mg\hat{i}\cdot d\vec{r} = \int_{x_A}^{x_B} \mu_d mg dx = \mu_d mg(x_B - x_A) = -\frac{1}{2}mv_x^2(0)$$

Secondo metodo

$$\vec{F}(t) = -\mu_d m g \hat{v} = \mu_d m g \hat{i} \quad \vec{v}(t) = \vec{v}(0) + \mu_d g t \hat{i}$$

$$\vec{F}(t)\cdot\vec{v}(t) = \quad \mu_d m g \hat{i}\cdot\left[\vec{v}(0) + \mu_d g t \hat{i}\right] \quad = \mu_d m g\left[-\left|v_x(0)\right| + \mu_d g t\right]$$

$$L_{attrito,A\to B} = \int_{t_A}^{t_B} \mu_d m g\left[-\left|v_x(0)\right| + \mu_d g t\right] dt = \mu_d m g\left[-\left|v_x(0)\right| t_B + \frac{1}{2}\mu_d g t_B^2\right]$$

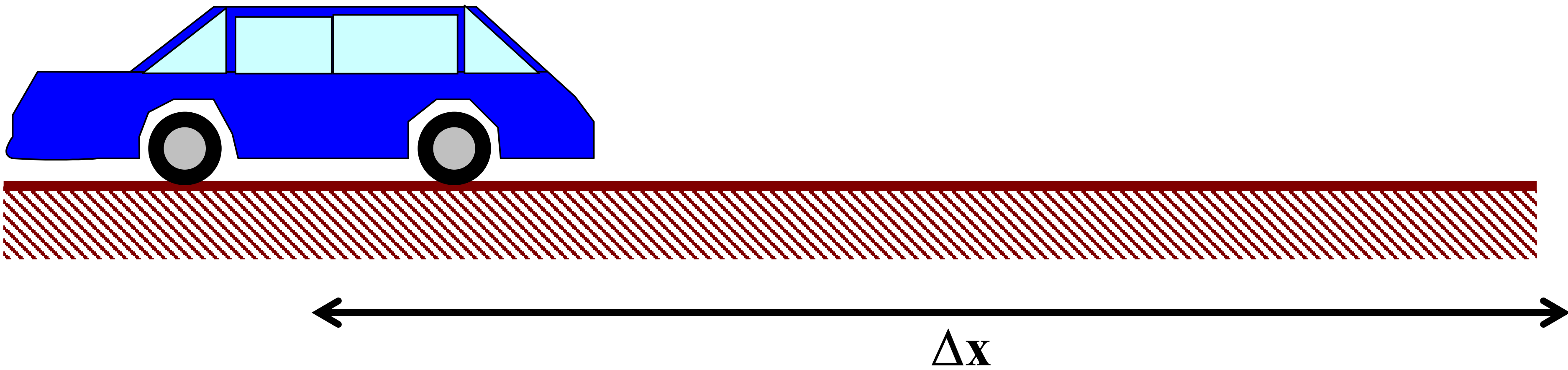
$$t_A = 0 \quad t_B = \frac{\left|v_x(0)\right|}{\mu_d g}$$

$$L_{attrito,A\to B} = -\mu_d m g\left[\frac{v_x^2(0)}{2\mu_d g}\right] = -\frac{1}{2} m v_x^2(0)$$

Frenata regolare: lo spazio di frenata dipende dall'energia cinetica

$$\vec{F}_{freni}(t) = -\gamma\hat{v}(t) \qquad \vec{F}_{freni}(t)\cdot\vec{v}(t) = -\gamma v(t)$$

(γ dipende dalla spinta sul pedale)



$$\frac{1}{2}mv^2(\text{finale}) - \frac{1}{2}mv^2(\text{iniziale}) = L_{attrito} = \int_{t_{iniziale}}^{t_{finale}} -\gamma\frac{dx}{dt}dt = -\gamma\Delta x$$

$$\frac{1}{2}mv^2(\text{finale}) \;\Vert\; 0$$

$$\frac{1}{2\gamma}mv^2(\text{iniziale}) = \Delta x$$

Esempio 2: forza di gravità

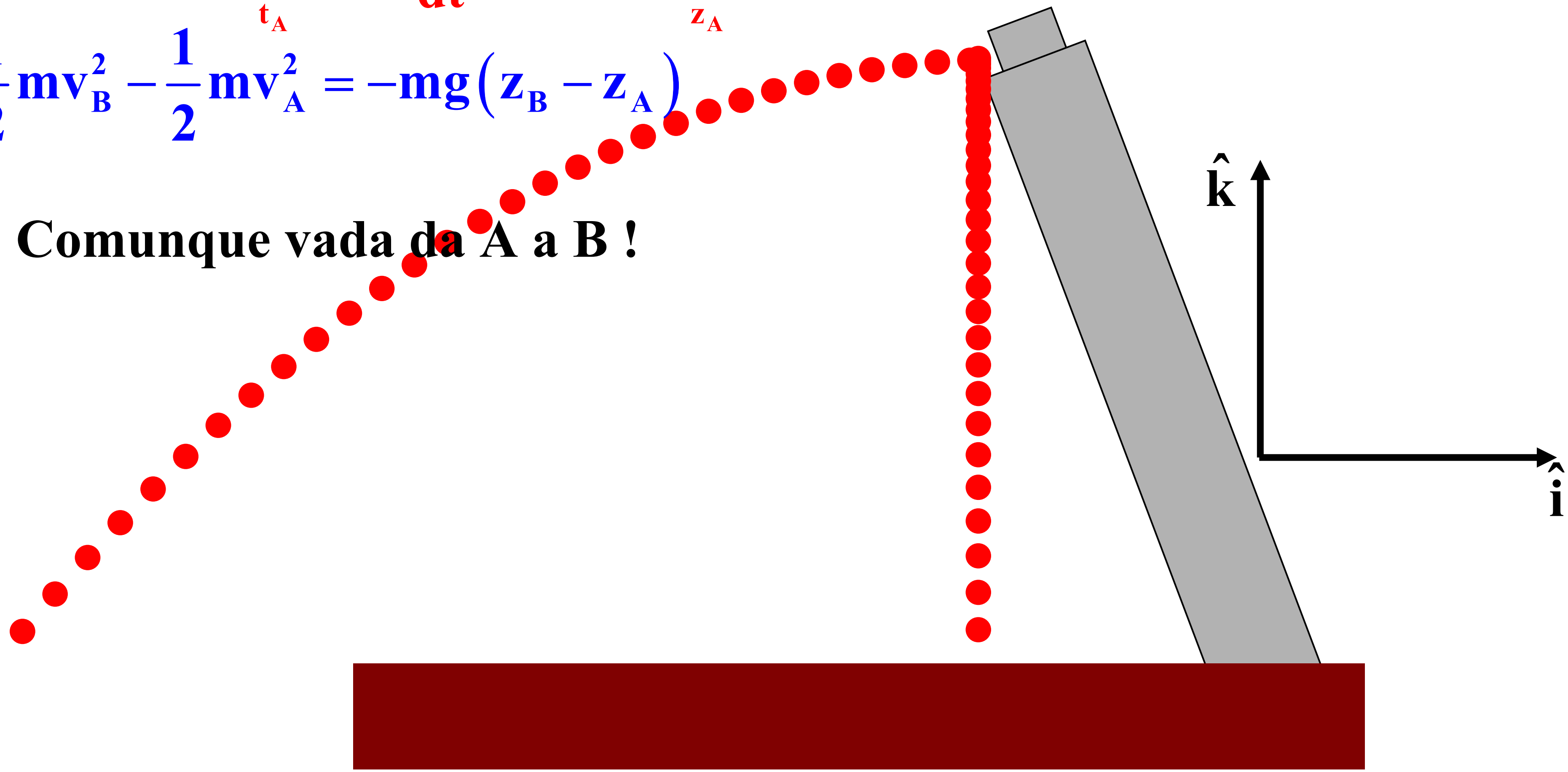
$$\vec{F}(t) = -mg\hat{k}$$

$$\vec{F}(t)\cdot\vec{v}(t) = -mg\hat{k}\cdot\vec{v}(t) = -mgv_z(t)$$

$$L_{gravità,A\to B} = \int_{t_A}^{t_B} -mg\frac{dz}{dt}dt = -mg\int_{z_A}^{z_B} dz = -mg(z_B - z_A)$$

$$\frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2 = -mg(z_B - z_A)$$

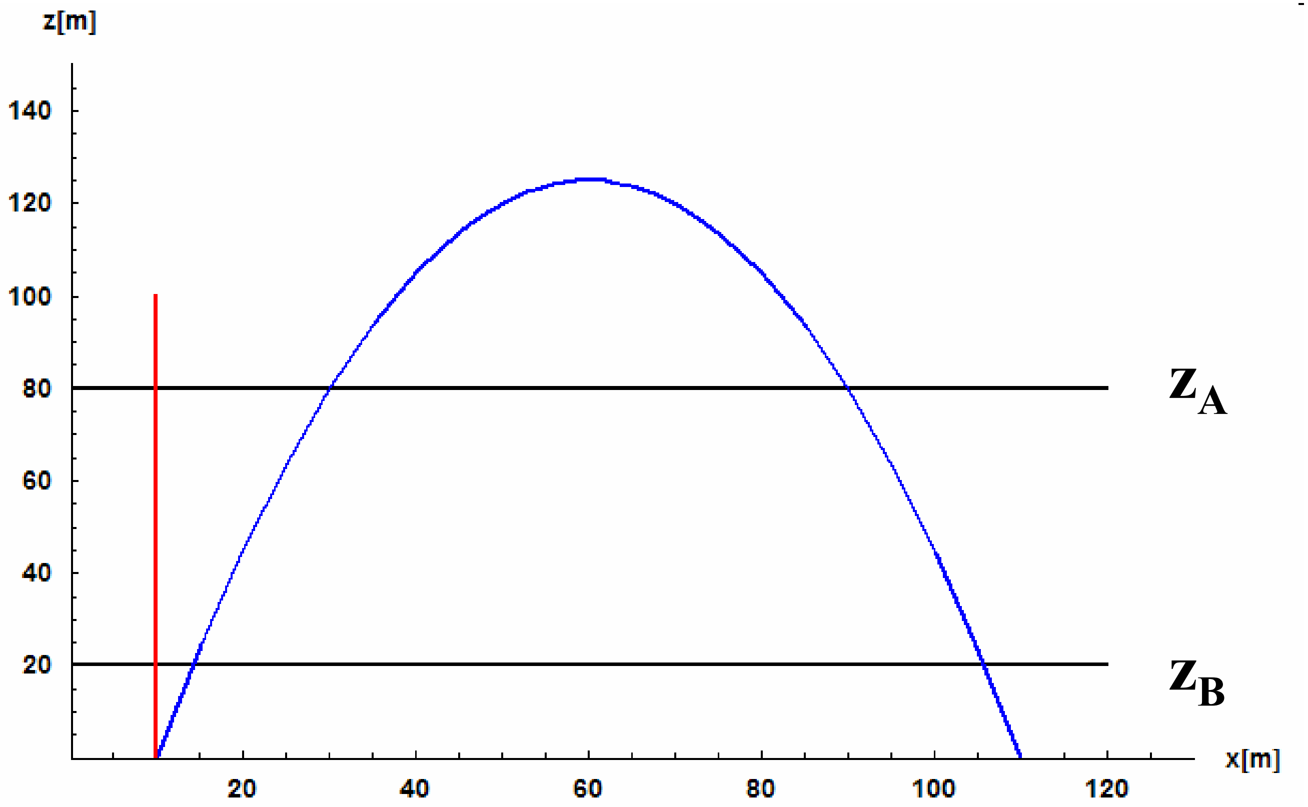
Comunque vada da A a B !



Controlliamo

$$z(t) = 100\text{m} - \frac{1}{2}gt^2 \quad x(t) = 10\text{m} \quad v_z(t) = -gt \quad v_x(t) = 0$$

$$z(t) = 50\frac{\text{m}}{\text{s}}t - \frac{1}{2}gt^2 \quad x(t) = 10\text{m} + 10\frac{\text{m}}{\text{s}}t \quad v_z(t) = 50\frac{\text{m}}{\text{s}} - gt \quad v_x(t) = 10\frac{\text{m}}{\text{s}}$$



z_A

z_B

Un'importante proprietà:

$$\frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2 = -mg\left(z_B - z_A\right)$$

$$\Downarrow$$

$$\frac{1}{2}mv_B^2 + mgz_B + C = \frac{1}{2}mv_A^2 + mgz_A + C$$

Definendo: L'energia potenziale $U(z) = mgz(+C)$

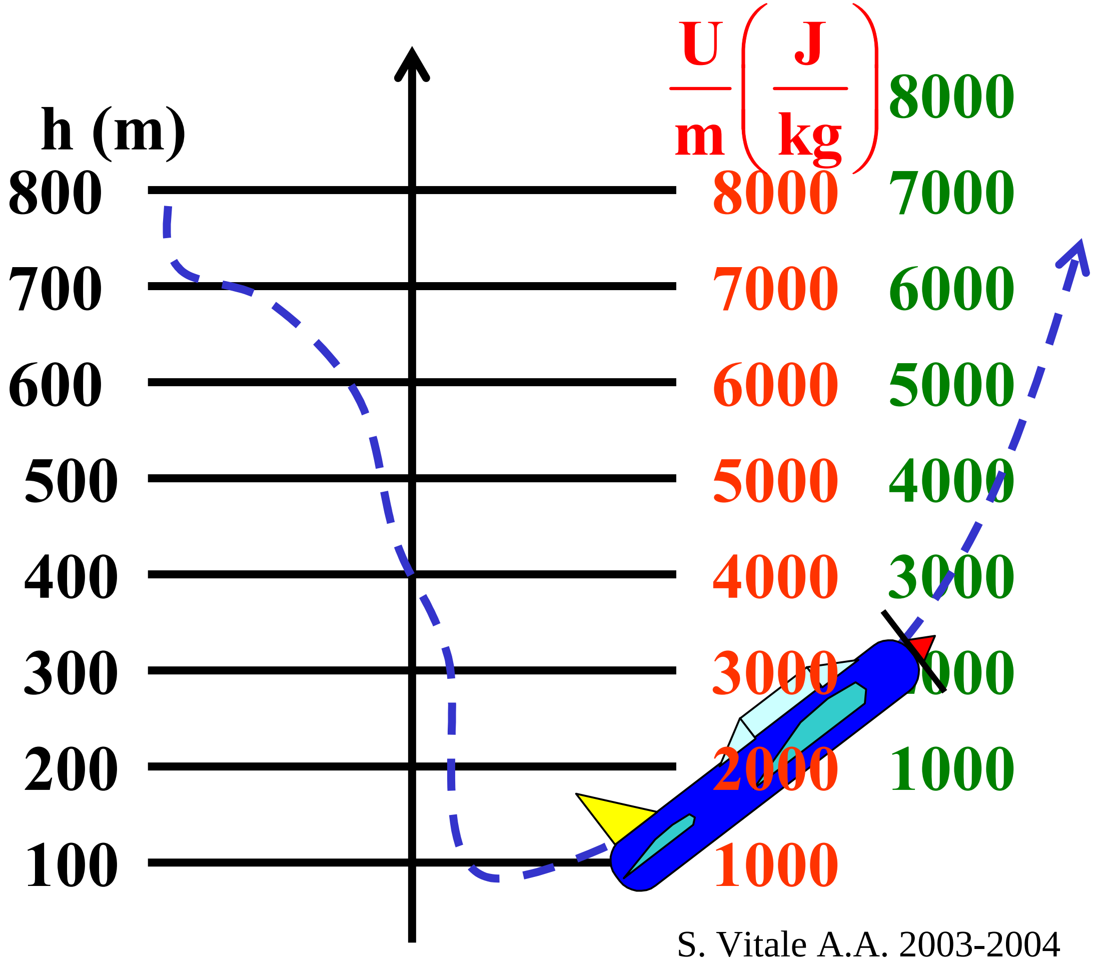
E l'energia meccanica totale $E = U + \frac{1}{2}mv^2$

Teorema di conservazione dell'energia $E_A = E_B$

L'energia potenziale:

1 Solo le differenze U_B-U_A contano

2 Perché potenziale?



Può essere sempre riconvertita in energia cinetica

La conservazione dell'energia più in genere. Se:

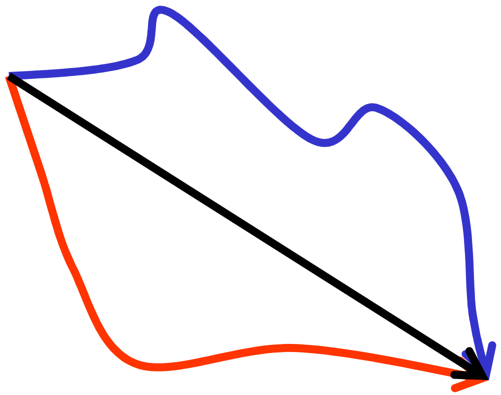
1 $\vec{F} = \vec{F}(x, y, z)$

(N.B. se: $\vec{F} = \vec{F}(x, y, z, t)$ campo di forze, se

$\vec{F} = \vec{F}(x, y, z)$ campo stazionario)

2 $L_{A \to B} = \int_A^B \vec{F} \cdot d\vec{r} = f(x_A, y_A, z_A, x_B, y_B, z_B)$

A

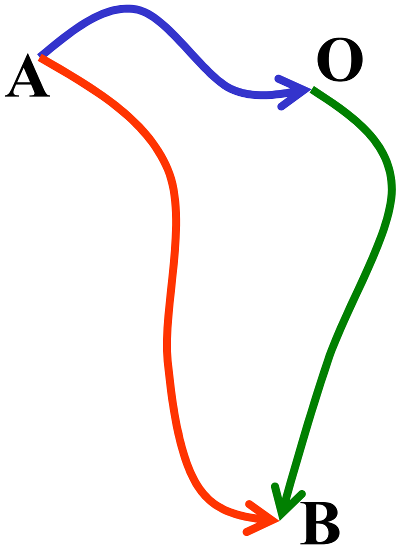


B

Cioè se:

$L_{A \to B} = L_{A \to B} = L_{A \to B}$

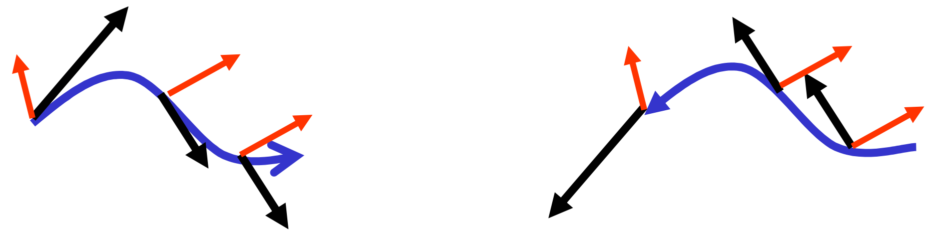
Il campo è conservativo



Se il lavoro non dipende dalla curva effettivamente seguita

$$L_{A\to B} = L_{A\to O} + L_{O\to B}$$

Ma se si inverte il verso di percorrenza della curva



$$\vec{F} \to \vec{F} \quad d\vec{r} \to -d\vec{r}$$

$$\vec{F}\cdot d\vec{r} \to -\vec{F}\cdot d\vec{r}$$

$$L_{A\to O} = -L_{O\to A}$$

$$L_{A\to B} = L_{O\to B} - L_{O\to A} \equiv V_O(B) - V_O(A)$$

Se su un punto materiale agisce solo una forza conservativa

(o una somma di sole forze conservative)

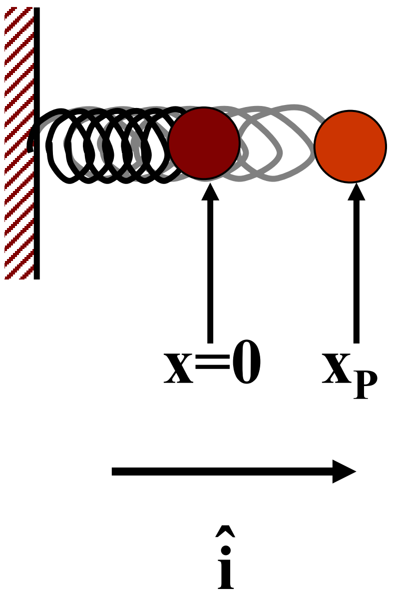
$$\left.\begin{aligned} L_{tot,A\to B} &= V_O(B) - V_O(A) \\ L_{tot,A\to B} &= \frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2 \end{aligned}\right\} \to \frac{1}{2}mv_B^2 - V_O(B) = \frac{1}{2}mv_A^2 - V_O(A)$$

$$E_B = \frac{1}{2}mv_B^2 + U_O(B) = \frac{1}{2}mv_A^2 + U_O(A) = E_A$$

$$\left[\text{Energia potenziale: } U_O(x) = -V_O(x)\right]$$

E l'energia meccanica $E = \frac{1}{2}mv^2 + U$ **si conserva**

Un'esempio semplice: campi unidimensionali:

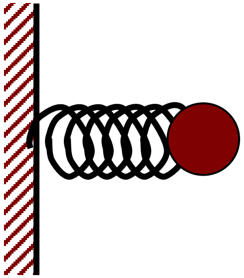


$$\vec{F} = -kx\hat{i}$$

$$L_{0\to P} = \int_0^{x_P} -kx\hat{i}d\vec{r} = \int_0^{x_P} -kxdx = -\frac{1}{2}kx^2\Big|_0^{x_P}$$

$$U_O(P) = -L_{O\to P} = \frac{1}{2}kx_P^2$$

N.B. un campo: $\vec{F} = f(x)\hat{i}$ **è sempre conservativo**



Ma se il campo è conservativo l'energia meccanica totale si conserva

$$m\frac{d^2x}{dt^2} = -kx \quad \rightarrow \quad m\frac{d^2x}{dt^2} + kx = 0$$

$$x(t) = x_c \mathrm{Cos}\left(\sqrt{\frac{k}{m}}t\right) + x_s \mathrm{Sin}\left(\sqrt{\frac{k}{m}}t\right) = x_o \mathrm{Cos}\left(\sqrt{\frac{k}{m}}t + \phi\right)$$

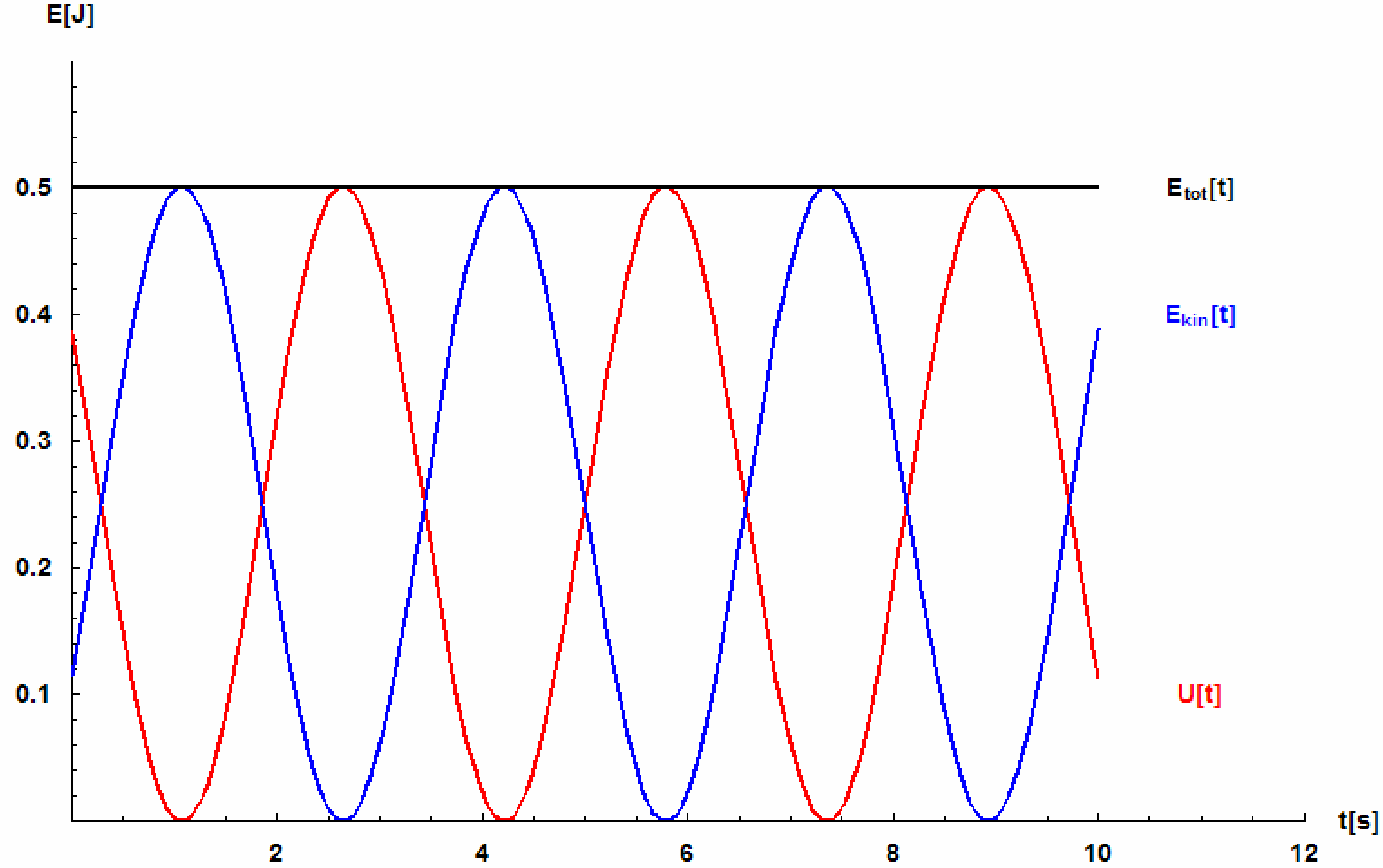
$$\left[x_o = \sqrt{x_c^2 + x_s^2} \quad \phi = \text{-Arc}\tan\left(\frac{x_s}{x_c}\right)\right]$$

$$v_x(t) = -x_o\sqrt{\frac{k}{m}}\mathrm{Sin}\left(\sqrt{\frac{k}{m}}t + \phi\right)$$

$$E = \frac{1}{2}mv^2(t) + \frac{1}{2}kx^2(t) =$$

$$= \frac{1}{2}m\frac{k}{m}x_o^2\mathrm{Sin}^2\left(\sqrt{\frac{k}{m}}t + \phi\right) + \frac{1}{2}kx_o^2\mathrm{Cos}^2\left(\sqrt{\frac{k}{m}}t + \phi\right) =$$

$$= \frac{1}{2}kx_o^2$$

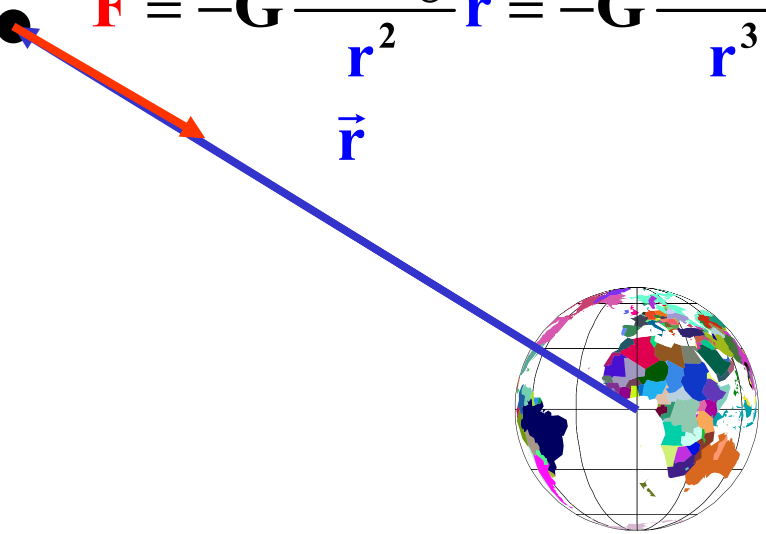


x_o=1m, k=1 N/m, m=1 kg, ϕ=0.5 rad

$$E = \frac{1}{2} k x_o^2 = 0.5\ 1 \frac{N}{m} (1m)^2 = 0.5J$$

Un esempio difficile: la gravitazione Newtoniana

$$\vec{F} = -G\frac{mM_{\oplus}}{r^2}\hat{r} = -G\frac{mM_{\oplus}}{r^3}\vec{r}$$



$$G = 6.67\times 10^{-11}\frac{m^3}{kg\ s^{-2}}$$

$$\vec{F}\cdot\vec{v} = -\frac{GmM_{\oplus}}{\left(x^2+y^2+z^2\right)^{3/2}}\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\cdot$$

$$\cdot\left(\frac{dx}{dt}\hat{i}+\frac{dy}{dt}\hat{j}+\frac{dz}{dt}\hat{k}\right) =$$

$$= -\frac{GmM_{\oplus}}{\left(x^2+y^2+z^2\right)^{3/2}}\left(x\frac{dx}{dt}+y\frac{dy}{dt}+z\frac{dz}{dt}\right)$$

$$\frac{d\left(x^2+y^2+z^2\right)}{dt} = 2\left(x\frac{dx}{dt}+y\frac{dy}{dt}+z\frac{dz}{dt}\right)$$

$$\vec{F}\cdot\vec{v} = -\frac{GmM_\oplus}{2\left(x^2+y^2+z^2\right)^{3/2}}\left(x\frac{dx}{dt}+y\frac{dy}{dt}+z\frac{dz}{dt}\right) =$$

$$= -\frac{GmM_\oplus}{2\left(x^2+y^2+z^2\right)^{3/2}}\frac{d\left(x^2+y^2+z^2\right)}{dt} =$$

$$= -\frac{GmM_\oplus}{2r^3}\frac{dr^2}{dt} \qquad = -\frac{GmM_\oplus}{r^2}\frac{dr}{dt} \qquad = GmM_\oplus\frac{d\,{}^1\!/\!_r}{dt}$$

$$U_O(P) = -L_{O\to P} = -\int_{t_O}^{t_P}\vec{F}(t)\cdot\vec{v}(t)\,dt = -\int_{t_O}^{t_P} GmM_\oplus\frac{d\,{}^1\!/\!_r}{dt}dt =$$

$$-\frac{GmM_\oplus}{r_P}+\frac{GmM_\oplus}{r_O}$$

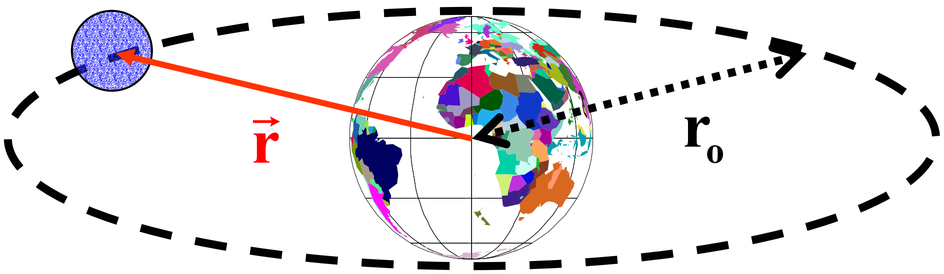
Prendendo il punto O a distanza infinita:

$$U_O(P) = -\frac{GmM_\oplus}{r_P} + \frac{GmM_\oplus}{r_O = \infty} = -\frac{GmM_\oplus}{r_P}$$

L'energia totale si conserva

$$\frac{1}{2}mv^2(t) - \frac{GM_\oplus m}{r(t)} = E_o = \text{Costante}$$

Esempio: orbita circolare



Moto circolare uniforme: $\vec{a} = -\omega^2 \vec{r}$ $\quad \vec{F} = m\vec{a} = -\omega^2 m \vec{r}$

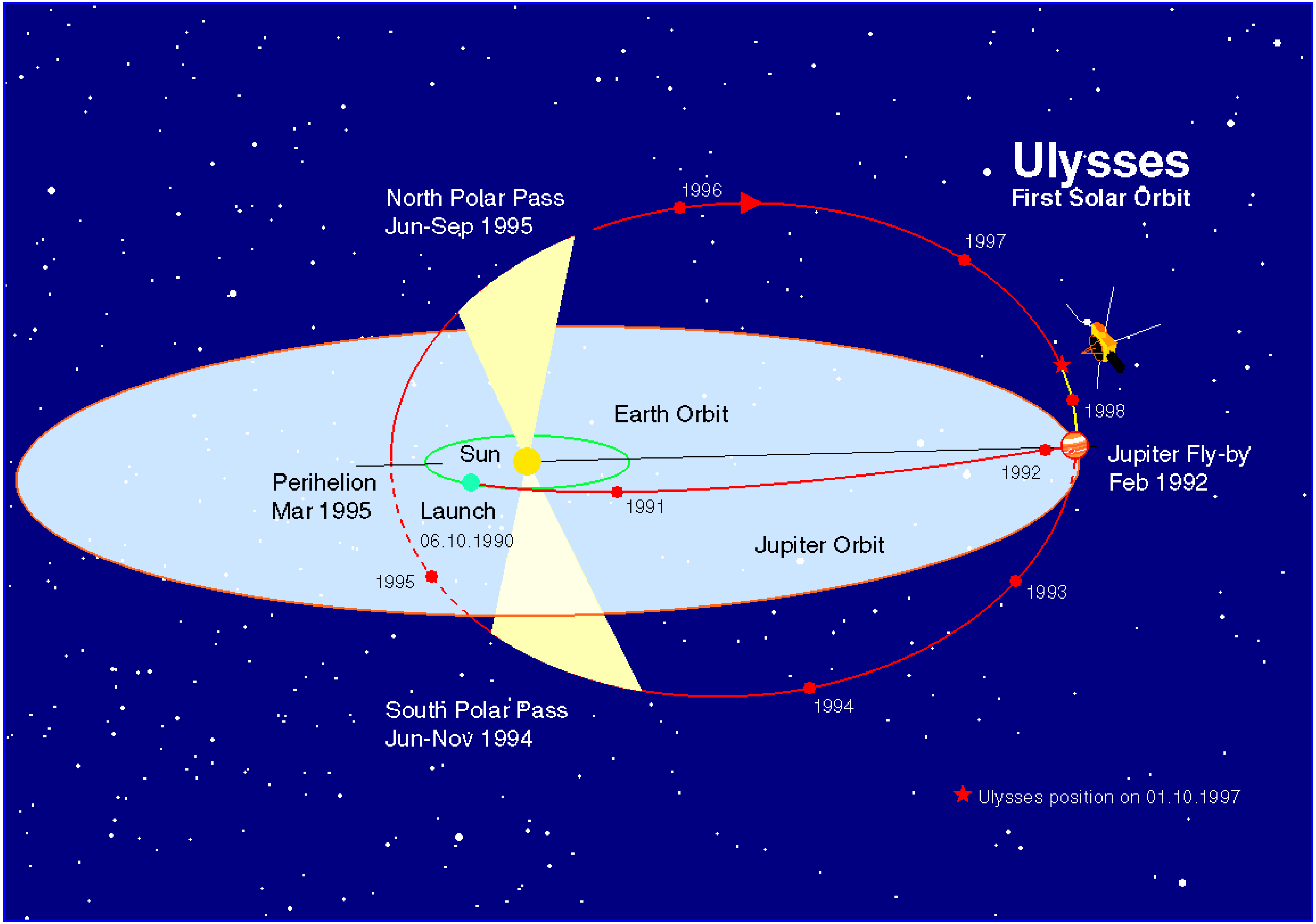
Se la gravità può fornire questa forza il moto circolare uniforme è possibile

$$-\frac{GM_\oplus m}{r^3}\vec{r} = -m\omega^2\vec{r}$$

$$\frac{GM_\oplus}{r_o^3} = \omega^2$$

Keplero: il quadrato del periodo è proporzionale al cubo della distanza

$$E = \frac{1}{2} m r_o^2 \omega^2 - \frac{GM_\oplus m}{r_o} = \cos t$$


Ulysses
First Solar Orbit
North Polar Pass
Jun-Sep 1995
1996
1997
1998
Earth Orbit
Sun
Perihelion
Mar 1995
Launch
06.10.1990
1991
1992
Jupiter Fly-by
Feb 1992
Jupiter Orbit
1995
1993
1994
South Polar Pass
Jun-Nov 1994
Ulysses position on 01.10.1997

Anno	x(AU)	y(AU)	z (AU)	r(AU)
1993.01171875	-4.588	1.614	-1.388	5.058
1993.01989746	-4.58	1.608	-1.4	5.052
1993.02807617	-4.571	1.601	-1.413	5.045
1993.03625488	-4.562	1.595	-1.425	5.039
1993.04443359	-4.553	1.589	-1.437	5.032
1993.0526123	-4.544	1.583	-1.449	5.026
1993.06079102	-4.535	1.576	-1.462	5.019
1993.06896973	-4.526	1.57	-1.474	5.012
1993.07714844	-4.517	1.564	-1.486	5.005
1993.08532715	-4.507	1.557	-1.498	4.999
1993.09350586	-4.498	1.551	-1.51	4.992
1993.10168457	-4.488	1.544	-1.522	4.985
1993.10986328	-4.479	1.538	-1.534	4.978
1993.11804199	-4.469	1.531	-1.546	4.971
1993.1262207	-4.459	1.525	-1.558	4.963

Anno	x(AU)	y(AU)	z (AU)	r(AU)
1995.25268555	1.227	-0.413	0.459	1.373
1995.26086426	1.219	-0.399	0.512	1.381
1995.26904297	1.211	-0.385	0.564	1.39
1995.27722168	1.201	-0.37	0.617	1.4
1995.28540039	1.19	-0.354	0.668	1.41
1995.2935791	1.178	-0.339	0.719	1.421
1995.30175781	1.164	-0.323	0.769	1.432
1995.30993652	1.15	-0.307	0.818	1.445
1995.31811523	1.135	-0.29	0.867	1.458
1995.32629395	1.119	-0.274	0.915	1.471
1995.33447266	1.101	-0.257	0.962	1.485
1995.34265137	1.083	-0.239	1.009	1.5
1995.35083008	1.064	-0.222	1.055	1.515
1995.35900879	1.045	-0.205	1.099	1.53
1995.3671875	1.024	-0.187	1.143	1.546

Velocità (AU/Anno):

1993

0.978149	-0.733612	-1.46722
1.10042	-0.855881	-1.58949
1.10042	-0.733612	-1.46722
1.10042	-0.733612	-1.46722
1.10042	-0.733612	-1.46722
1.10042	-0.85588	-1.58949
1.10042	-0.733612	-1.46722
1.10042	-0.733612	-1.46722
1.22269	-0.855881	-1.46722
1.10042	-0.733612	-1.46722
1.22269	-0.855881	-1.46722
1.10042	-0.733612	-1.46722
1.22269	-0.855881	-1.46722
1.22269	-0.733612	-1.46722
1.22269	-0.855881	-1.46722

1995

-0.978149	1.71176	6.48024
-0.978149	1.71176	6.35797
-1.22269	1.83403	6.48024
-1.34496	1.9563	6.2357
-1.46722	1.83403	6.2357
-1.71176	1.9563	6.11343
-1.71176	1.9563	5.99116
-1.83403	2.07857	5.99116
-1.9563	1.9563	5.86889
-2.20084	2.07857	5.74663
-2.20084	2.20084	5.74663
-2.3231	2.07857	5.62436
-2.3231	2.07857	5.37982
-2.56764	2.20084	5.37982
-2.56764	2.20084	5.37982

1993		1995	
$\frac{1}{2}v^2\left(\frac{m^2}{s^2}\right)$	$-\frac{GM_\oplus}{r}\left(\frac{m^2}{s^2}\right)$	$\frac{1}{2}v^2\left(\frac{m^2}{s^2}\right)$	$-\frac{GM_\oplus}{r}\left(\frac{m^2}{s^2}\right)$
8.20862´ 10^{7}	-3.50777´ 10^{8}	1.03247´ 10^{9}	-1.29223´ 10^{9}
1.00589´ 10^{8}	-3.51194´ 10^{8}	9.97146´ 10^{8}	-1.28474´ 10^{9}
8.78053´ 10^{7}	-3.51681´ 10^{8}	1.05434´ 10^{9}	-1.27642´ 10^{9}
8.78053´ 10^{7}	-3.521´ 10^{8}	1.00186´ 10^{9}	-1.26731´ 10^{9}
8.78053´ 10^{7}	-3.5259´ 10^{8}	9.99164´ 10^{8}	-1.25832´ 10^{9}
1.00589´ 10^{8}	-3.5301´ 10^{8}	9.93109´ 10^{8}	-1.24858´ 10^{9}
8.78053´ 10^{7}	-3.53503´ 10^{8}	9.59803´ 10^{8}	-1.23899´ 10^{9}
8.78053´ 10^{7}	-3.53997´ 10^{8}	9.80661´ 10^{8}	-1.22784´ 10^{9}
9.85707´ 10^{7}	-3.54492´ 10^{8}	9.47353´ 10^{8}	-1.21689´ 10^{9}
8.78053´ 10^{7}	-3.54917´ 10^{8}	9.49374´ 10^{8}	-1.20614´ 10^{9}
9.85707´ 10^{7}	-3.55415´ 10^{8}	9.61149´ 10^{8}	-1.19477´ 10^{9}
8.78053´ 10^{7}	-3.55914´ 10^{8}	9.30535´ 10^{8}	-1.18282´ 10^{9}
9.85707´ 10^{7}	-3.56414´ 10^{8}	8.69979´ 10^{8}	-1.17111´ 10^{9}
9.41973´ 10^{7}	-3.56916´ 10^{8}	9.08667´ 10^{8}	-1.15963´ 10^{9}
9.85707´ 10^{7}	-3.57492´ 10^{8}	9.08667´ 10^{8}	-1.14763´ 10^{9}

1993 — Valori medi

$$\frac{1}{2}v^2\left(\frac{m^2}{s^2}\right) \qquad -\frac{GM_\oplus}{r}\left(\frac{m^2}{s^2}\right)$$

$$9.24255\times 10^{7} \qquad -3.54027\times 10^{8}$$

1995

$$\frac{1}{2}v^2\left(\frac{m^2}{s^2}\right) \qquad -\frac{GM_\oplus}{r}\left(\frac{m^2}{s^2}\right)$$

$$9.66285\times 10^{8} \qquad -1.22489\times 10^{9}$$

Energia Totale: $\frac{1}{2}mv^2 - \frac{GM_\odot m}{r}$

1993: $-2.62\ 10^{8}$ J/kg   1995: $-2.58\ 10^{8}$ J/kg